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ANGLE COMPARATORS OF HIGH PRECISION FOR THE GONIOMETRY OF PRISMS

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ABSTRACT. This paper describes applications of the method of substitution to goniometric measurements of high precision. A simple arrangement is described with which measurements accurate to within one or two seconds can readily be made and a more elaborate arrangement with which an accuracy of about 0·1 second may be attained. The latter embodies a method, which appears to be new, of obtaining very minute variations in the direction of a beam of light emerging from a collimator by placing near the focal plane of the latter a "variable prism" of simple design and construction.

WHEN it is required to measure the angles of prisms to a high order of accuracy, the familiar methods all cease, for various reasons, to be satisfactory. In the first place there are not more than a few spectroscopes or goniometers in the country on which readings can be taken to an accuracy of, say, a second of arc; and even with such instruments the greatest care is required, combined with the use of suitable methods, before an accuracy of this order can be attained in the final result.

Alternative methods, obviating the use of an accurate goniometer, have usually therefore been utilised by those requiring to make precise angle measurements. Of these, methods employing interference fringes are the most satisfactory and the most free from unsuspected sources of error. The drawback to the interference method is the time involved and the accuracy of adjustment required. This rules out the method for anything but standardisation work in which cost is not a primary object. Many methods employing an auto-collimating telescope have also been described by various workers. In most cases these suffer from a quite serious

source of possible error, as pointed out by the present writer*, and are not suitable for work of high accuracy.

Experiments made by the writer some years ago showed that, as in most other classes of measurement, the simplest, quickest, and most accurate method of determining angles was the *method of substitution*, i.e. the method in which the minute difference between an angle under test and a standard of the same nominal value is measured. The method is clearly only applicable to cases where such a standard is available, but, as we shall see presently, standards of 90°, 60°, 45°, and 30°, which are the important angles as far as optical work is concerned, can readily be evaluated, and are then available for general testing.

A Simple Angle Comparator.

A simple arrangement by which measurements to an accuracy of a second or thereabouts can be made is illustrated in Fig. 1. C is a collimator and T_1 and T_2 are telescopes, the latter being provided with a screw micrometer eyepiece. S is a prism support provided with the three adjustment screws common to spectroscope tables, and provided also with a tangent screw giving a short range of fine adjustment in rotation. C, T_1, T_2, and S should be mounted on rigid stands placed on an iron table or a slate slab with a flat surface. It is necessary that the optical axes of the collimator and telescopes should be approximately in the same plane. This adjustment may be made as follows. One of the telescopes is placed in line with the collimator and adjusted until the centre of the slit, which should be indicated by a cross fibre in the usual way, is at the same height in the field of view as the intersection of the cross-lines. This telescope is then removed and the other adjusted in the same way. It is clear that if the collimator axis is parallel to the surface of the table so also, after the adjustment just described, are those of the two telescopes. If, however, the collimator is inclined so that the emergent light slopes, say, upwards, the optical axis of each telescope now slopes downwards (from eyepiece to objective) by the same amount. If we place the telescopes in line, and place a lamp behind the eyepiece of one of them, we shall see an image of its cross-lines in the field of the other. This will be higher or lower than the cross-lines of the latter by twice the amount, in angular measure, by which the axis of each telescope is inclined to the table. Half the adjustment required to bring the cross-lines into coincidence is made on each telescope, thus bringing both axes into parallelism with the table.

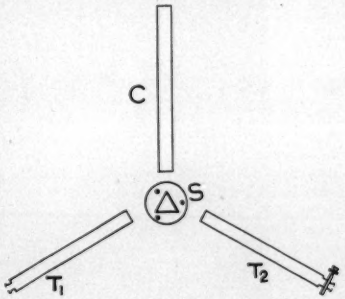

Fig. 1.

* *Proc. Phys. Soc.* 28 (1916), 242.

Either telescope may then again be directed into the collimator and the latter adjusted until the centre of the slit comes on the cross-wires. By this procedure all three axes are adjusted parallel to the table on which the parts rest, and so are coplanar in any position.

In making a measurement one of the nearly equal angles to be compared is placed on the prism table and the telescopes T_1 and T_2 are placed to receive the light reflected from the two faces, as shown in Fig. 1, which illustrates the arrangement for a 60° angle. The prism is adjusted by means of the levelling screws of the table until both images appear at the proper height. The image in T_1 is then brought exactly to the cross-lines by means of the tangent screw which rotates the prism table. The image in T_2 is obtained in a convenient part of the field by moving the telescope bodily by hand; the cross-lines are then set on the image by operating the eyepiece micrometer. The first angle is then removed from the prism table and the second one placed as nearly as possible in the same position. The adjustment for height of the images having been carried out as before, the image in T_1 is set exactly on the cross-wires by the tangent screw of the table. If the two angles were equal, the image in T_2 would now also be on the cross-wires. In general, however, this is not so, and the micrometer must be reset. The difference between the micrometer settings in the two cases gives, when converted to angular measure, twice the difference in the angles.

The whole process is very simple and can be carried out in a few minutes. The chief precaution which must be taken, in order that the conditions may be exactly the same for both angles, is to see that central portions of the apertures of the telescopes T_1 and T_2 are employed in each case. This can be secured by mounting iris diaphragms in front of the objectives and closing them to such dimensions that they are completely filled by the beams which enter them. If this precaution is taken the measurements will be entirely free from any errors arising from aberrations of the lenses or slight changes of focus impressed on the beams by any lack of perfect flatness of the prism surfaces.

It will be noted that no assumption is made of parallelism of the beams emerging from the opposite halves of the collimator lens. The two beams are separate entities as far as this method is concerned, and could have been obtained from separate collimators if need be. There is not, therefore, any source of error due to imperfect adjustment of the collimation as in the ordinary rotating telescope method of measurement on a spectrometer or the auto-collimating methods criticised in the paper already referred to. Needless to say no part of the apparatus, except the prism table adjustments and the micrometer eyepiece, may be touched between the first and last observations.

With reasonable care it is possible by this method to make measurements correct to about a second of arc, provided a good quality micrometer eyepiece be used with telescopes giving a magnification of about 20 diameters. An alternative arrangement, which obviates the use of a collimator, is to fit the telescopes T_1 and T_2 with auto-collimating eyepieces. They are then placed normal to the surfaces bounding the angle to be measured, the procedure being in all other respects identical with that

already described. The main disadvantage of this plan is that it is not easy to arrange an entirely satisfactory auto-collimating device in conjunction with a screw micrometer eyepiece.

Establishment of Standards.

The establishment of standard angles for use in the substitution method of test is readily accomplished in the case of all the important angles by means of the method itself. The simplest case is that of a 60° standard. This should consist of an equilateral prism polished on all three sides and free from pyramidal error*. The differences between the angles, two at a time, are measured; their sum being known, the actual values follow at once. For 90° the standard prism should be of square section, the procedure being the same as for 60°. For 45° the standard should be a 90°, 45°, 45° prism. The right angle is determined by comparison with a 90° standard (see above), from which the sum of the two 45° angles is obtained. Their difference is determined by substitution. For 30° the standard should be a 90°, 60°, 30° prism. The first two angles are determined by comparison with the appropriate standards, the difference between their sum and 180° giving the remaining 30° angle.

With a set of standards thus established for all the common angles, the subsequent testing of prism angles of these nominal values is very simple. A large number can be compared with a standard in a comparatively short time, the results having an accuracy which it is difficult to attain by direct measurements, even on the most elaborate goniometers.

It is, of course, assumed that the prisms are of sufficiently good quality as regards flatness of surface that their angles can be specified to high accuracy. If the surfaces are not flat the angle between them varies from place to place, and a different "mean" value will be obtained by different methods of test. It is clearly meaningless to express the value of an angle to the nearest second if the angle varies by several seconds at different parts. But whatever the quality of the prism, the most accurate result that can be obtained for it is given by the substitution method more easily than by any other of which the writer is aware. The method has been used with consistent success at the National Physical Laboratory for the last seven years for work of all degrees of precision.

An Improved Angle Comparator: General principles.

While an accuracy of one second or thereabouts is ample for most practical purposes, it was felt desirable from the point of view of a standardising institution to explore the possibilities of still greater precision in such measurements. Experiments showed that the accuracy attainable with arrangements involving micrometer eyepieces was limited, not by the capabilities of the eye, but by the mechanical imperfections of the micrometers, and that for a given focal length of observing telescope considerably greater precision would be possible with a theoretically perfect micrometer than with any actual specimen of the instrument-makers' art.

* For the effect of pyramidal error on these measurements see Guild, *Trans. Opt. Soc.* 22 (1920–21), 139.

Experiments were therefore made with a micrometric device in which no screw is involved, and which is of such a nature that mechanical defects would have to be very gross in order to affect the measurements to an appreciable extent. The device consists of a variable prism of very small angle placed between the telescope objective and its focal plane, and near the latter. If θ is the deviation produced by the prism and l is its distance from the focal plane of the telescope $l\theta$ is the displacement of an image formed by the objective from the position which it would occupy in the absence of the prism. If θ can be varied, we have the means of producing movement of the image and can bring it to coincidence with fixed cross-wires. Suppose that the direction of the light before entering the telescope changes by a small angle $\delta\phi$. The image in the focal plane will move a distance $f\delta\phi$ where f is the focal length of the objective. In order to bring the image back to the fixed cross-lines we must alter θ by such an amount, $\delta\theta$, that $l\delta\theta = f\delta\phi$ or $\delta\theta = (f/l)\,\delta\phi$. If f/l is large, that is, if the variable prism is near the focal plane, it requires a comparatively large change in θ to compensate a small change in the direction of the incident light. Hence, if the deviation of the prism is small and can be varied with certainty to within a small fraction of itself, we have a very precise means of compensating, and therefore of measuring, minute changes in the direction of the incident light.

Conversely, if we regard the telescope as a collimator, and illuminate the cross-lines, or slit or other suitable object in their place, from behind, a given change in θ will produce an apparent change in the angular position of the object, as seen through the objective, of $(l/f)\,\delta\theta$. If l/f is small we can produce very minute changes in the direction of the light leaving the collimator by suitably varying θ.

There are two well-known types of variable prism. One of these consists of two equal prisms of small angle, rotatable in opposite directions. The deviation varies from zero, when the two components are deviating in opposite directions, to twice the deviation due to either when both are deviating in the same direction. The other type consists of two weak lenses, of equal focal length but of opposite sign, one of which is provided with a motion of translation in front of the other. When the axes of the lenses are coincident, the combination acts as a plane parallel plate, but when the axes are relatively displaced by a distance x, say, the combination acts as a wedge-shaped plate, deviating the light by an angle x/F where F is the focal length of the component lenses. This latter type was adopted for the present purpose because one previously made for other work happened to be available. Apart from this, however, it is the more convenient form for incorporation in the apparatus presently to be described, though the other form could no doubt be used and might even be preferable in some modifications of the apparatus.

The two component lenses of the wedge in question were of approximately 8 metres focal length; both were rectangular, the positive, which is fixed in position, being 3 cm. long by 2·5 cm. deep and the negative 10·5 cm. long by 2·5 cm. deep. The latter was mounted in a frame hung from a steel rod 6 mm. in diameter and 46 cm. long which was supported on two frictionless pulleys at the extremities of a suitable bracket. This permitted the lens to be moved freely over

a range a little in excess of its own length. The fixed lens was mounted at the centre of the bracket supporting the movable lens and just in front of the latter. A millimetre scale was attached to the movable lens and a vernier, reading to 0·1 mm., to the bracket.

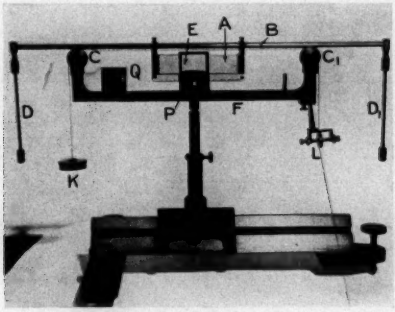

Fig. 2.

This part of the apparatus is shown in Fig. 2. A is the movable lens suspended from the rod B which simply rests on the freely working pulleys C and C_1. D and D_1 are weight rods fitting friction tight on the ends of the supporting rod. Their function is to lower the centre of gravity of the moving system and overcome any tendency to move out of the vertical plane which the suspended lens may exhibit when the supporting rod rolls along on the pulleys. E is the fixed lens, held in a mount attached to the supporting bracket, F. There is a clearance of about 1 mm. between the moving and fixed lenses. A millimetre scale is attached to the moving lens and a vernier plate, in the same plane as the scale, is mounted just below it at the centre of the supporting bracket. This vernier is not seen in the photograph, being concealed by the 45° prism P, which, in conjunction with another 45° prism Q, is employed to facilitate reading the scale from a distance, as described later. A clearance of about 0·2 mm. is left between the lower edge of the scale plate and the upper edge of the vernier plate, as it is undesirable that the moving part should be in rubbing contact with any part of the apparatus. The bracket is supported on a short length of solid optical bench of the type originally made by Zeiss and now manufactured in this country.

It is necessary to operate this part of the apparatus from a distance, and the method first tried, and finally adopted after many intermediate experiments with more pretentious arrangements, was to rotate the pulleys C and C_1, by means of a stout thread passing over them. This thread supports a weight K, passes over the pulleys C and C_1, passes round an auxiliary pulley, L, which is attached to the bracket by a universal joint, so that it can be turned in any direction, and is then

led to any point convenient to the observer. The thread is not attached to the moving lens system, but passes through holes in the lens supports. The micrometer is operated by means of a drum round which the thread is coiled. By winding or unwinding the thread the weight K is raised or lowered, and the moving lens is carried in whichever direction is required. The form of drum used in the present apparatus is simply a screw of 6 mm. diameter working in a nut of adjustable tightness. The thread is attached to and coiled round the free part of the screw. The latter has a milled head of about 2·5 cm. diameter, and, when the tightness of the nut is adjusted properly, the whole forms a very sweetly working micrometer. It might be thought that the elasticity of the thread would militate against the smooth working of such a device as that just described, but this is not so. The material actually employed is silk fishing line of a medium thickness; and, although there is a length of six feet between the operating screw and the pulley L, the moving lens responds to the slightest movement of the screw, and the reading of the scale and vernier may be adjusted with precision by amounts as small as 0·1 mm. It may be mentioned that this very excellent result, though easily attainable with the simple rolling rod suspension described, was by no means attained by several more pretentious methods of suspension which were tried at various times.

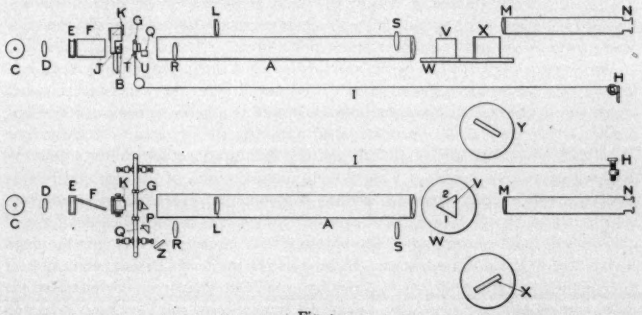

Fig. 3.

Having described the "variable prism" micrometer and the method of its operation we may now consider the complete apparatus. This is shown in diagrammatic plan and elevation in Fig. 3. To avoid confusion non-essential parts have mostly been omitted. A collimator A, consisting of an astronomical telescope, of about 125 cm. focal length and 7·5 cm. aperture, from which the eyepiece fittings have been removed, is mounted on V supports on a triangular section optical bench of the same pattern as, but longer than, that already described. On a third support, clamped to the same bench, is a plate B, situated in the focal plane of the collimator lens. In this plate are cut a vertical slit about 1 mm. wide and 6 or 7 mm. long, and a square hole of 6 mm. side, separated from the slit by about 2·5 mm.

The slit is provided with a longitudinal fibre and two transverse fibres*, separated by about 0·5 mm. to mark the approximate midpoint. The vertical fibre serves as the object. It is illuminated by means of a Pointolite lamp C, and a lens D, which forms an enlarged image of the luminous ball on the slit. It is an advantage to provide the lens D with an iris diaphragm to restrict the angular aperture of the beam so that it is only very little greater than the angular aperture of the collimator. This reduces stray light to a minimum. A cell E, containing a solution of cupric chloride, is interposed in the path of the beam to absorb the heat rays and prevent damage to the fibre. F is an inclined glass plate the purpose of which will be described later.

Light diverging from the object is collimated by the object glass of the collimator. The variable prism device is mounted at G a short distance from the object. By means of the screw H, which winds up or lets out the thread I, the wedge can be operated, and the direction of the beam emerging from the collimator altered by very small amounts as already explained.

If the emergent beam ultimately meets a surface exactly normal to it, it will be reflected back through the objective of the collimator and an image of the object will be formed coincident with the object itself. This would not, however, be very easy to observe, so instead of causing the image to fall on the object it is arranged to fall on the adjacent square hole, which is provided with diagonally mounted cross-wires of the same thickness as the object fibre.

It is not convenient to observe the field of view directly by means of an eyepiece. In the first place the observer's head would be in the way of the incident light, and in the second place the observer would be unable to reach the levelling screws which have to be operated when adjusting the apparatus. To overcome these difficulties an auxiliary optical system is employed to view the focal plane B from a convenient position. A small right angled prism, of the size and quality used as spectroscope comparison prisms, is mounted at J behind the square hole which constitutes the field of view, care being taken that it does not cut off any of the cone of light converging to the object slit. A larger prism, K†, is mounted above this, in the position shown. Light reaching the field of view passes on, *via* the prisms J and K, to the lens L, which again collimates it. An observing telescope, MN, is mounted in line with the axis of the system JKL, and the observer sees in the field of this telescope the field of view of the collimator at B. No cross-lines are fitted in the observing telescope, the system $JKLMN$ merely serving to view the field at B from a convenient vantage point. The lens L requires to be a good quality telescope objective similar to M. Both lenses must have angular apertures slightly in excess of the angular aperture of the main collimator. In the apparatus as set up the lenses L and M are identical; it will be seen, therefore, that the image formed in the focal plane of MN is of unit magnification, and the final power of

* The fibres should be between 10 and 20μ thick. Quartz fibres or thick spider web may be employed, but fibres, suitable for cross-wires of all kinds, can be drawn from glass with very simple apparatus. They are very robust and stand considerable shock.

† A good quality prism is required here.

the whole system is the same as if the eyepiece N had been used to view the field at B directly.

In order that the scale and vernier of the variable wedge may be read without the observer having to change his position the prisms P and Q are fitted on little platforms attached to the bracket carrying the wedge (see also Fig. 2). These, in conjunction with the lenses R and S, enable the observer to see a slightly magnified image of the scale and vernier with his left eye without moving his right eye from the eyepiece of the observing telescope. The scale and vernier are diagrammatically represented at T and U. The first lens, R, forms an image of these at or near the second lens S, and this lens in turn forms an image of the aperture of R in the plane of the observer's eye. With the eye anywhere within the area of this latter image the whole of the field comprised within the boundary of the lens S is visible. The lenses should be of such focal length that the image of the vernier formed near S is slightly smaller than the diameter of S, so that the whole vernier is within the field of view. There is, however, considerable latitude in the powers of the lenses which may be adjusted to give satisfactory conditions. The lenses actually used consisted of a telescope lens of 35 cm. focal length and 4 cm. diameter at S, while two similar lenses were combined to form the lens R. This procedure was adopted not because of any particular virtue in the use of a double lens but because it happened to be the simplest method of getting the desired power with the material available.

In using the apparatus for the comparison of two nearly equal angles, one of them is placed at V on a table W provided with three adjusting screws as ordinarily found on spectrometer tables. The table is also provided with a tangent screw adjustment for rotation about its axis. Two plane mirrors X and Y are mounted on exactly similar tables. It is desirable that the normals to these mirrors and the axis of the collimator A should be approximately coplanar. This adjustment is made as follows: A plane parallel glass plate is mounted on the prism table and levelled up until the image reflected from it is at the same height in the field of view, in both of the positions, 180° apart, in which the plate reflects light directly back into the collimator. The stands supporting the latter are now adjusted so that the midpoint of the image of the slit, halfway between the transverse fibres, is at the same height as the cross-lines. The optical axis of the collimator is now perpendicular to the axis of rotation of the prism table. The latter is now rotated so that light reflected from the optical flat meets the mirror X, which is adjusted to reflect it normally back so that after a second reflection at the flat it re-enters the collimator. X is levelled up so that the image comes to the proper height in the field. Y is then adjusted in the same way. If, as in the present apparatus, the axes of rotation of the prism and mirror supports are accurately perpendicular to the surface of the table on which the apparatus is assembled, the adjustment once made will be correct in whatever position the mirrors X and Y may be, as required for measuring angles of various sizes. In any case the adjustment only requires to be very approximate, as the error introduced by a departure from the ideal conditions is negligible in a substitution method though vital in any measurement of the

total angle. When the prism is in position at V the beam of light leaving the collimator is split into two parts by reflection at its faces. The beam reflected from face 1 meets the mirror X normally, is again reflected by 1 and re-enters the collimator, an image of the slit and vertical fibre being formed in the focal plane. By rotating X this image is brought on or near the cross-wires. The mirror Y is similarly adjusted so that the beam originally reflected from face 2 of the prism also comes back to form an image coincident with or close to that from face 1. The prism is levelled up in the usual way so that both of these images are at the proper height in the field, that is to say, so that the point halfway between the images of the transverse fibres is level with the intersection of the cross-wires.

The exact angular separation of the images is now measured by bringing first one and then the other to the cross-wires by operating the variable prism G. If the images are close together or overlapping it is a convenience to cut out one of them when setting on the other, by interposing a screen between the prism and the appropriate mirror X or Y. Having made this measurement for the first angle, this is removed from the table and the second angle put in its place, care being taken to set it as nearly in the same position as possible. This is best accomplished by attaching suitable guides to the table top. By means of the tangent screw of the prism table the prism is rotated until the images appear in the field of view and the prism is levelled up as before. The separation of the images is then measured as in the case of the first angle. Since the mirrors X and Y have not been touched during adjustment of the second angle any difference in the separation of the images from that obtained in the first case is due to a difference in the angles, being, in fact, equal to twice that difference.

Practical Requirements.

It will at once be obvious that in order to measure angles to the order of a second of arc, or fractions thereof, by means of a built-up apparatus the greatest rigidity in all the parts is essential, and the operations involved in making the measurements must not require any of the vital parts to be touched by the operator. A photograph of the actual apparatus as constructed at the Laboratory is shown in Fig. 4, from which an idea of the essential features can be gathered. The lettering corresponds to that of Fig. 3. The base of the apparatus consists of a cast iron table, 6 feet by 3 feet, of the type used by engineers for setting out work. No wooden table, however robust, is satisfactory. A slate slab resting on a three-point support on a wooden table, or built to a wall, would no doubt serve the purpose, but a solid metal table, if available, cannot readily be improved on, short of employing concrete piers built into the ground.

It will be noted that the short length of optical bench which carries the variable wedge arrangement, G, bridges over and *does not touch* the three-foot length of similar bench which carries the collimator A and the cross-lines. This is important, as it ensures that any slight deformation of the wedge support which may occur under the pull of the operating string is not transmitted to the parts of the collimator, as would undoubtedly happen if the wedge were supported from the same

bench. The great beauty of the variable wedge as a micrometer lies in the fact that the reading is not affected by any lateral movements of the wedge apparatus as a whole relative to the collimator, but depends simply on the relative positions of the two lenses constituting the wedge, as shown by the scale and vernier. This enables the parts which require to be moved during measurement to be mounted quite separately from those parts which define the optical axis of the system; the latter, therefore, remain immune from any strains imposed during operation. The screw *H*, on which the string is wound, has been mounted on a separate stand, a large shell gauge. This is almost a superfluous refinement: a suitable support resting on the table can be used quite well; but the separate stand was employed because it was available, and it does give the advantage that the hand or forearm may be rested on it without introducing any deformation of the table. Needless to say the table itself should never be leant upon. The adjustable tables on which the prism under test, *V*, and the mirrors, *X* and *Y*, are mounted are of a rigid type. As a matter of interest they have all been constructed from scrapped munition gauges of various kinds.

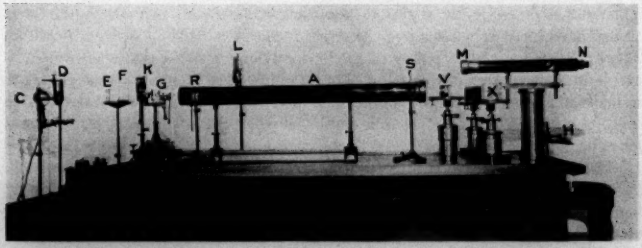

Fig. 4.

It will be observed from a study of Fig. 3 that the only thing which requires to remain constant during the complete comparison of two angles is the horizontal angle between the normals to the reflecting faces of the mirrors *X* and *Y*, and brief consideration will serve to show that it is extremely difficult to disturb this angle by means of any feasible flexure of the supporting table, temperature changes, or any shock likely to arise from vibration of the building. The constancy of direction of the optical axis of the collimator, though important, is only of secondary importance compared with the inclination of *X* and *Y*. The actual separation of the images is not affected by the direction of the beam leaving the collimator, but only their position in the field of view. Hence it is only necessary that the direction of the beam should remain constant during the short time involved in bringing first one and then the other image to the fixed cross-lines. It may change during the substitution and adjustment of the second angle without affecting the result. It will thus be seen that the design of the arrangement gives the maximum con-

stancy where it is most required. Experiments showed that the inclination of X and Y remained constant to within 0·2 second for a period of several days, although the optical axis of the collimator varied within a range of over a second in this time.

Rigidity of a high order is not required in the systems employed for observing the field of view and for reading the scale. Any convenient type of lens stand can therefore be used to support the lenses L, R, and S, while any reasonably steady stand serves for the telescope MN.

The scale and vernier must be well illuminated. For obvious reasons the number of light sources employed should be a minimum, and should be situated on the axis of the apparatus to prevent asymmetrical heating. It was found that good scale illumination could be secured by diverting some of the light from the Pointolite lamp by means of the inclined plate F and reflecting this by means of a concave mirror Z (Fig. 3) so as to pass obliquely through the prism P (same figure) to the scale and vernier, which are of white ebonite. The mirror Z was removed before taking the photograph, Fig. 4, to avoid confusion. The mirrors X and Y must be of the very best quality. Speculum or glass silvered on the front may be used. Those actually employed are large interferometer mirrors* by Messrs Adam Hilger. Though not perfectly flat they have almost exactly similar contours, and this is all that is required. The telescope objectives L and M must be of very good quality. Those shown are Cooke lenses of 50 cm. focal length and 5 cm. aperture. The eyepiece N usually employed is of about 13 mm. focal length, so that as this is virtually used directly with the collimator A (focal length 125 cm.) the magnification is 96. A power of half this or less is employed when the prism under test does not give good enough definition for such magnification.

There is one point which it is desirable to mention in connection with setting up such an apparatus. It may be found that the field of view is flooded with stray light. This may be due to flare spot images of some of the apertures. Such images are usually very small and can easily be located and blocked out by a thick wire or other suitable stop. Another possible source of stray light in the field is reflection by the surfaces of the lenses comprising the variable wedge. This can be got over by very slightly inclining the plane of this part of the apparatus so that the reflected light does not enter the field of view.

Results.

The wedge employed consists of two lenses of + and − 8 metres focal length. The deviation produced by this combination when the axes of the components are displaced by 1 mm. is about 26 seconds. By placing the wedge about 5 cm. in front of the focal plane of the collimator lens (focal length = 125 cm.), a displacement of 1 mm. corresponds to $26 \times 5/125$ secs., or approximately 1 second change in direction of the beam emerging from the collimator. This corresponds to 0·5 sec. in the angle under test. The scale can be read easily to 0·1 mm., so that the least count of the measuring gear corresponds to about a tenth of a second in the position of the emergent beam, or a twentieth of a second in the measured angle. The scale

* Since replaced by speculum flats prepared in the Laboratory Optical workshop.

can of course be made either more or less open than this by placing the wedge nearer to or further from the focal plane; but experience showed that about 1 mm. per second was a convenient arrangement for precise work. The scale must of course be calibrated in angle. This is done by mounting a telescope with micrometer eyepiece directly in line with the collimator, and measuring the angle which the emergent beam moves through when the movable lens of the variable wedge passes from one end of its range to the other. The total range as set up is about 75 seconds, which with care can be determined to an accuracy of about half a second, employing a good micrometer eyepiece which has itself been calibrated over a still larger range on a suitable goniometer.

For a range of angle not exceeding ten seconds, therefore, the calibration error need not exceed a tenth of a second. It may be thought that this is a very minute range. This defect is common to all precise differential methods. It is only necessary, however, that such a method should cover the range within which the type of constant which it is intended to measure may reasonably be expected to vary, and it is quite possible to bring any angle to within a few seconds of its nominal value before it leaves the workshop. It is rarely indeed that a prism which is otherwise of the very best quality has angles departing from the intended values by more than 10 seconds, while for standardisation purposes they can be brought much nearer than this if desired.

The sensitivity of settings, as distinct from the accuracy of final results, was tested by taking long series of settings on one image, adjusted for best focus. The mean difference from the mean of such series naturally varied from time to time with the condition of the observer, but the highest value obtained did not exceed 0·1 second in the direction of the emergent beam, while a value as low as 0·03 second was once reached. As regards sensitivity of setting, there is therefore no difficulty in obtaining values, under optimum conditions of definition, within a tenth of a second for the emergent beam, i.e. within a twentieth second in the angle measured. There are, however, difficulties in the way of obtaining this accuracy in the final result of an angle measurement. These do not reside in the measuring apparatus. It would be impossible to misread the scale by more than 0·1 second, and there is no other source of error which can come into the measurements, all errors due to aberrations of the optical parts being eliminated in the substitution method. The difficulties are associated with the quality of the surfaces of the prisms under test. Any departure from flatness of the faces not only makes it impossible to define the angle between them with such accuracy, but also, by affecting the focus of the reflected beams, impairs the definition and so destroys the precision of the actual observations. Angle standards intended for the most precise measurements must be as perfect as the optician can possibly make them, the remarks made under this head when describing the simpler form of apparatus applying with tenfold force when measurements of the enhanced accuracy here contemplated are under consideration. All measurements must of course be made with the prism at a reasonably steady temperature; if the temperature is either rising or falling the surfaces, if originally flat, will take on considerable curvature.

The existing angle standards at the National Physical Laboratory are by no means perfect, not having been intended for measurements of this kind. Despite the fact that the definition given by them when used with this instrument is very poor, it has been found possible to establish the values of some of them to within ± 0·12 second. New standards are being prepared in the optical workshop of the Laboratory which, it is hoped, will enable more accurate values to be obtained. They are being made of such dimensions that the full aperture of the collimator will be employed.

Usefulness of Method.

The fact that an accuracy of the order of 0·1 second in a prism angle is meaningless and unattainable except with prisms of exceptionally accurate construction, and then only in suitable conditions as to steadiness of temperature, may render it difficult to see any practical utility in measurements carried to this extreme. There are several important purposes, however, for which such accuracy is not superfluous.

1. *Standardisation work.* As already mentioned, the measurement of angles to within 1 or 2 seconds, which is frequently required in good optical work, is greatly facilitated by comparing with suitable standards. Such standards ought clearly to be known with certainty to a small fraction of the error which is material in the desired result. Angle standards which can be measured and checked to within a small fraction of a second are much more satisfactory for use in measurements of one second accuracy than standards which may themselves be in error by this amount. Not only so but routine measurements of ordinary precision can be carried out more quickly and with greater confidence on apparatus capable of much higher precision. Thus while considerable care is required to obtain an accuracy of a second on the simple apparatus first described, it would scarcely be possible, short of very gross carelessness, to make an error as large as a second with the improved form.

2. *Refractometry.* There are many problems of scientific and technical interest in connection with refraction and dispersion which require for their solution measurements of refractive index to the sixth decimal place. The prism angle must be known in some cases to within 0·2 second for such work. Usually this measurement may be avoided altogether by using a prism polished on all three sides, and making refractive measurements with all three angles in turn and taking the mean. Such procedure is not in general possible with crystalline substances for which the refractive index varies in different directions, and the angle may have to be determined. The present method provides a means of doing so with ample accuracy, thereby removing one from the number of formidable difficulties which have to be overcome in sixth decimal place refractometry.

THE CHANGES IN ABERRATIONS WHEN THE OBJECT AND STOP ARE MOVED

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ABSTRACT. If the aberrations of any centered optical system are known both for an object which intersects all rays transmitted by the system and also for the centre of the effective stop, the position in the image space of the emergent portion of a given incident ray is known, and the aberrations in the image of any other object for any stop position can be expressed in terms of those for the first object. The investigation aims to express the relations in the second case in terms of those present in the first when the objects are planes normal to the axis of symmetry whatever the order of the aberration may be.

The investigation is based upon a convenient potential function, and the Eikonal, with the principal foci as reference points, is chosen as the standard function. First a distinction is drawn between aberrational and non-aberrational terms, and the discussion leads to the general form taken by the sine-condition in the outer parts of the field of view; it also becomes evident that there is only one stop aberration of each order to be considered, and that this is of spherical aberration form. The transformation is achieved by a change of variables, and to secure simple forms the aberrations of any order are arranged in series, each series forming a separate group as regards transformations. The rays in a plane containing the axis belong entirely to one series, which is called the zero series; in this series the coefficients in general must have finite values depending on the position of the object and of the stop if the aberrations are to be removed. In all the other series the coefficients should be zero for freedom from the aberrations, the conditions thus being independent of the positions of the stop and object. As the number of the series increases the expressions for the calculation of the coefficients become more simple. These facts are illustrated by the conditions for first aberrations, all of which are members of series o except the Petzval sum which belongs to series 1.

The formulae obtained enable the aberrations to be expressed in terms of the coefficients of the standard Eikonal or *vice versa*, or alternatively the aberrations for any object and stop positions to be given in terms of those for standard positions. For use in the latter case certain advantages are secured by choosing for the stop the position where the magnification is + 1 and for the object the surface for which the magnification is − 1.

As a very simple application the principles underlying the use of the formulae to obtain with the minimum of labour the effects of small variations in the constructional data of the system are illustrated.

As the paths of rays are fully known when their intersections with two planes are given, it is possible with any optical system to express the aberrations present for any assigned positions of the image and of the stop in terms of those present when these occupy any other positions. The equations representing these transformations have a number of important applications. To mention only one, aberrations for

any positions may be evaluated in terms of those of any other positions which are regarded as standards, the latter usually being chosen so as to give symmetrical forms or alternatively so that a number of the aberrations will necessarily be zero or at least very small, thus reducing the number of terms which require independent evaluation to a minimum. When the selected positions for standard purposes correspond to the magnifications $+1$ and -1 the constants may be used, with at most a change of sign in certain of them, for the system in its direct and in its reversed form. This reference system is found to be much the most convenient for thin and approximately thin lenses, and assumes particular importance in the theory of close lenses, from which almost all real systems are constructed.

Although the transformations to be investigated can be derived without difficulty by direct geometrical reasoning, there are many advantages in basing the enquiry on the properties of one of the potential functions which have been introduced into the theory of optical instruments. One of the chief advantages of this course is that any assumptions which violate the laws of light propagation are automatically avoided, and at the same time all aberrations are displayed in the lowest terms to which they can be reduced.

As the Eikonal has been extensively employed by the author as the foundation of a symbolic system for aberrations which offers the fullest symmetry, this function will be employed in the present paper. The form of the relations found is independent of the choice of function, though the precise interpretation of the aberrations, when finite, varies slightly with the different functions available.

Let then $\mathscr{E}$ denote the length of the optical path along a particular ray between the feet of the perpendiculars to the incident and emergent portions of the ray from the principal foci for the object and image spaces respectively. The direction cosines of the incident ray will be represented by L, M, N and those of the emergent ray by L', M', N', the axis of x in either case being coincident with the axis of symmetry. It is convenient to assume that the refractive index of both external media is unity. This involves no loss of generality for the form in which these refractive indices enter into the expression for $\mathscr{E}$ is known and they may if desired be inserted into the final result. Alternatively it is possible to define the aberration coefficients in a form into which these indices do not enter. The Eikonal $\mathscr{E}$ may be regarded as a function of the four direction cosines M, N, M', N', and the intersections of the ray with the two paraxial focal planes are then given by the equations

$$\left.\begin{aligned} y &= \frac{\partial \mathscr{E}}{\partial M}, & z &= \frac{\partial \mathscr{E}}{\partial N} \\ y' &= -\frac{\partial \mathscr{E}}{\partial M'}, & z' &= -\frac{\partial \mathscr{E}}{\partial N'} \end{aligned}\right\} \qquad \text{.....(1).}$$

If the points of intersection with planes normal to the axis distant x and x' from the principal foci are desired, both x and x' being positive when following the direction in which light passes through the system from the object space to the image space, it is only necessary to replace $\mathscr{E}$ in (1) by $\mathscr{E}'$ where

$$\mathscr{E}' = \mathscr{E} - xL + x'L' \qquad \text{.....(2),}$$

L and L' being of course regarded as functions of M and N and of M' and N' respectively. In particular if the intersections of the ray with the planes through the conjugate points for magnification G are desired

$$\mathscr{E}_G\kappa = \mathscr{E}\kappa - \frac{L}{G} - L'G \qquad \text{.....(3),}$$

where κ is the power of the system for paraxial rays. If the instrument is a telescope it will be noted that no investigation can be based upon the direction cosines as they fail to identify a ray uniquely, so that no additional limitation is involved in assuming that κ is finite.

The terms representing aberrations in the image.

$\mathscr{E}_G$ will consist of two parts, one of which expresses the variation in the length of the path between the perpendiculars from the axial points of the conjugate planes due to the obliquity of the ray, while the other indicates the presence of path variations which are due to aberrations in the image. It is thus necessary to consider which terms belong to the one part and which to the other in order to have a clear definition of aberrations.

From symmetry $\mathscr{E}$ is a function of (1), (2), and (3) where

$$(1) = M^2 + N^2, \quad (2) = MM' + NN', \quad (3) = M'^2 + N'^2 \qquad \text{.....(4),}$$

and as $L = \sqrt{1 - (1)}$, $L' = \sqrt{1 - (3)}$, $\mathscr{E}_G$ is a function of the same variables. Introduce new variables I, II, III defined by

$$\left.\begin{aligned} \mathrm{I}\,(S-G)^2 &= (1) - 2G\,(2) + G^2\,(3) \\ \mathrm{II}\,(S-G)^2 &= (1) - (S+G)\,(2) + SG\,(3) \\ \mathrm{III}\,(S-G)^2 &= (1) - 2S\,(2) + S^2\,(3) \end{aligned}\right\} \qquad \text{.....(5),}$$

so that

$$\left.\begin{aligned} (1) &= S^2\mathrm{I} - 2SG\mathrm{II} + G^2\mathrm{III} \\ (2) &= S\mathrm{I} - (S+G)\,\mathrm{II} + G\mathrm{III} \\ (3) &= \mathrm{I} - 2\mathrm{II} + \mathrm{III} \end{aligned}\right\} \qquad \text{.....(6),}$$

S being a number different from G.

The intersections of the ray with the conjugate planes are then given by

$$\left.\begin{aligned} y\,(S-G)^2 &= (M - GM')\,(2\mathscr{E}_\mathrm{I} + \mathscr{E}_\mathrm{II}) + (M - SM')\,(\mathscr{E}_\mathrm{II} + 2\mathscr{E}_\mathrm{III}) \\ y'\,(S-G)^2 &= (M - GM')\,(2G\mathscr{E}_\mathrm{I} + S\mathscr{E}_\mathrm{II}) + (M - SM')\,(G\mathscr{E}_\mathrm{II} + 2S\mathscr{E}_\mathrm{III}) \end{aligned}\right\} G\,(7),$$

where differentiation of $\mathscr{E}$ with regard to I, II, III is denoted by the addition of the appropriate suffix, and the G preceding the number of the equations means that the suffix G is to be understood to apply to all the $\mathscr{E}$'s in the equations. Two similar equations give z and z' if N and N' are substituted for M and M'. From these equations

$$\left.\begin{aligned} (y' - Gy)\,(S-G) &= (M - GM')\,\mathscr{E}_\mathrm{II} + 2\,(M - SM')\,\mathscr{E}_\mathrm{III} \\ \text{and similarly} \\ (z' - Gz)\,(S-G) &= (N - GN')\,\mathscr{E}_\mathrm{II} + 2\,(N - SN')\,\mathscr{E}_\mathrm{III} \end{aligned}\right\} \qquad \text{.....}G\,(8).$$

When aberrations are absent $y' - Gy = z' - Gz = 0$ for all rays, and as the relation

$$\frac{M - GM'}{N - GN'} = \frac{M - SM'}{N - SN'}$$

is only true for rays which are contained in a plane through the axis, it follows that in the absence of aberrations

$$\mathscr{E}_{II} = \mathscr{E}_{III} = 0 \qquad \ldots\ldots G\ (9),$$

or since the three variables I, II, and III are quite independent, $\mathscr{E}_G$ must be a function of I only. In other words, any term which involves II or III corresponds to an aberration, and when $\mathscr{E}_G$ is expressed as an infinite series the number of independent aberrations in the general case is the number of terms in which II or III are present. The coefficients of these terms, if desired with numerical coefficients also, will measure the magnitudes of the aberrations and may for convenience be referred to as aberrations themselves.

The extended sine law.

It may be noted in passing that since in the absence of aberrations $\mathscr{E}_G$ must be a function of I only, all the rays which start from a given point of the object plane must be refracted so that $M - GM'$ and $N - GN'$ have constant values expressible in terms of the coordinates y and z of this point, as is clearly seen from (7) on putting $\mathscr{E}_{II} = \mathscr{E}_{III} = 0$. This is the form the sine condition takes for the outer parts of the field of view, and is a particular case of a very general law, which the author has called the cosine law, from which the behaviour of rays in the neighbourhood of any given ray may be determined whatever the system may be and whether or not aberrations are present.

Aberrations of the stop.

When there are no aberrations in the image $\mathscr{E}_G$ is an arbitrary function of I. If this function is expressed in a series of ascending powers of I the first power depends only on the focal length of the system, and the other terms may be given such values that any ray passing through the axial point of the object plane corresponding to the magnification S is refracted through the axial point of the corresponding image plane. Departures from these values will evidently correspond to the presence of spherical aberration of various orders in the image of this plane. The real plane corresponding to the magnification S is assumed to be the plane of the aperture stop of the system, and it is necessary to add this one aberration to those previously enumerated to obtain a complete system. It will be noted that the presence of aberrations in the outer parts of the image of the stop is inevitable, as at once follows from the obvious result that if two pairs of planes have a one to one correspondence an infinite number of pairs of planes can be found satisfying the same relation.

To find the form which $\mathscr{E}_I$ must have for there to be no aberration at the centre

of the stop, the planes corresponding to the magnification S should be taken for reference. Thus

$$\mathscr{E}_S\kappa = \mathscr{E}\kappa - \frac{L}{S} - L'S$$

$$= \mathscr{E}_G\kappa + (S-G)\left(\frac{L}{SG} - L'\right)$$

and the coordinates (η, ζ), (η', ζ') of intersection of the ray with these planes, when $\mathscr{E}_G$ is a function of I only, satisfy

$$\left.\begin{aligned}\eta\,(S-G)^2 &= 2\,(M-GM')\,\mathscr{E}_{\mathrm{I}} - \frac{(S-G)^3}{SG\kappa}\cdot\frac{M}{L}\\ \eta'\,(S-G)^2 &= 2G\,(M-GM')\,\mathscr{E}_{\mathrm{I}} - \frac{(S-G)^3}{\kappa}\cdot\frac{M'}{L'}\end{aligned}\right\} \quad \ldots\ldots G\,(10),$$

with similar equations for ζ and ζ'. If $\eta = 0$ and $\eta' = 0$ are to occur simultaneously the rays concerned must satisfy

$$\frac{M}{L} = \frac{2SG\,(M-GM')}{(S-G)^3}\,\mathscr{E}_{\mathrm{I}}\kappa = S\frac{M'}{L'} \qquad \ldots\ldots G\,(11),$$

the equality of the first and third quantities expressing the tangent law. Eliminating M and M' from these equations gives

$$(SL - GL')\,2G\mathscr{E}_{\mathrm{I}}\kappa = (S-G)^3$$

and since $\mathscr{E}_{\mathrm{I}}$ is a function of I only it is only necessary to find the form $SL - GL'$ takes when it is a function of I alone.

If $S^2 = 1$ the solution takes particularly simple forms for evidently

$$M - SM' = N - SN' = L - L' = 0,$$

and

$$\mathscr{E}_{\mathrm{I}} = \frac{(S-G)^2}{2G\kappa}\,(1-\mathrm{I})^{-\frac{1}{2}},$$

or

$$\mathscr{E}_G = -\frac{(S-G)^2}{G\kappa}\,(1-\mathrm{I})^{\frac{1}{2}}.$$

When $S^2 \neq 1$ the equation for $\mathscr{E}_{\mathrm{I}}$ is a biquadratic which may be solved by successive approximation. The solution in series form may be written

$$\begin{aligned}\frac{G\kappa}{(S-G)^2}\,\mathscr{E}_G = {} & \tfrac{1}{2}\mathrm{I} + \tfrac{1}{8}e_1\,\mathrm{I}^2 - \tfrac{1}{16}\,(e_2 - 2e_1^2)\,\mathrm{I}^3 + \tfrac{1}{128}\,(5e_3 - 24e_2e_1 + 24e_1^3)\,\mathrm{I}^4\\ & - \tfrac{1}{256}\,(7e_4 - 40e_3e_1 - 18e_2^2 + 132e_2e_1^2 - 88e_1^4)\,\mathrm{I}^5\\ & + \tfrac{1}{1024}\,(21e_5 - 140e_4e_1 - 120e_3e_2 + 520e_3e_1^2\\ & \qquad + 468e_2^2e_1 - 1456e_2e_1^3 + 728e_1^5)\,\mathrm{I}^6 - \ldots \qquad \ldots\ldots(12),\end{aligned}$$

where $e_n = \dfrac{S^{2n+1} - G}{S - G}$. No constant term need be inserted as its value is quite immaterial. This is the value $\mathscr{E}_G$ must have when the system is to be regarded as fully corrected, and if this expression is subtracted in the general case from $\mathscr{E}_G$, all the terms which remain express aberrations. In the transformation formulae

which are to be found the inclusion of such terms as those given in (12) would be inconvenient, and consequently the formulae relate directly to the terms in the expansion of $\mathscr{E}_G$ without the subtraction of this part. It has then simply to be remembered that the coefficient of the term in I only of any order must have a correction applied to it if it is required to determine the aberration at the centre of the stop.

The values of the constants in $\mathscr{E}$ for which freedom from aberrations is secured for the two magnifications G, S obviously follows by substituting from (12) in

$$\mathscr{E} = \mathscr{E}_G + \frac{1}{\kappa}\left(\frac{\sqrt{1-(1)}}{G} + \sqrt{1-(3)}\,G\right),$$

and writing for I its value

$$\frac{(1) - 2G(2) + G^2(3)}{(S-G)^2}.$$

The denominators introduced by this substitution are most simply absorbed into the e's, changing e_n say to e'_n where

$$e'_n = \frac{S^{2n+1} - G}{(S-G)^{2n+1}}.$$

Thus the terms in $\mathscr{E}$ of the first three orders, when aberrations are absent are

$$\left.\begin{aligned}\mathscr{E}\kappa = &-(2) - \frac{1}{8}\left[\frac{(1)^2}{G} + (3)^2 G - \frac{e_1'}{G}\{(1) - 2G(2) + G^2(3)\}^2\right] \\ &- \frac{1}{16}\left[\frac{(1)^3}{G} + (3)^3 G + \frac{e_2' - 2e_1'^2}{G}\{(1) - 2G(2) + G^2(3)\}^3\right]\end{aligned}\right\} \quad \ldots\ldots(13).$$

Such a formula is in effect a particular case of the transformations it is desired to establish, and shows how the aberrations for G and S may be eliminated by giving the correct values to the various terms in the standard function $\mathscr{E}$.

The general transformation expressions.

For the general case it is desired to exhibit the coefficients for G', S' in terms of those for G and S. In place of I, II, III the aberrations for G' and S' are defined in terms of the variables I′, II′, III′ where by analogy with (5)

$$\left.\begin{aligned}\mathrm{I}'\,(S'-G')^2 &= (1) - 2G'(2) + G'^2(3) \\ \mathrm{II}'\,(S'-G')^2 &= (1) - (S'+G')(2) + S'G'(3) \\ \mathrm{III}'\,(S'-G')^2 &= (1) - 2S'(2) + S'^2(3)\end{aligned}\right\} \quad \ldots\ldots(14),$$

and thus by the solution corresponding to (6)

$$\begin{aligned}\mathrm{I}\,(S-G)^2 &= \mathrm{I}'\,(S'-G)^2 - 2\mathrm{II}'\,(S'-G)(G'-G) + \mathrm{III}'\,(G'-G)^2 \\ \mathrm{II}\,(S-G)^2 &= \mathrm{I}'\,(S'-G)(S'-S) \\ &\quad - \mathrm{II}'\,\{(S'-G)(G'-S) + (G'-G)(S'-S)\} \\ &\quad + \mathrm{III}'\,(G'-G)(G'-S), \\ \mathrm{III}\,(S-G)^2 &= \mathrm{I}'\,(S'-S)^2 - 2\mathrm{II}'\,(S'-S)(G'-S) + \mathrm{III}'\,(G'-S)^2.\end{aligned}$$

These relations may be expressed in a more concise way by a slight extension of the well-known notation in which crossed brackets are used, the factors after the

intersecting brackets being expanded before the coefficients in the preceding bracket are applied. For instance

$$(a, b, c \,|\, x, x')\,(y, y')$$

is used to indicate

$$axy + b\,(x'y + xy') + cx'y'.$$

In this notation the equations become

$$\left.\begin{aligned} \mathrm{I} &= (\mathrm{I}', \mathrm{II}', \mathrm{III}' \,|\, 1 + s, -g)^2 \\ \mathrm{II} &= (\mathrm{I}', \mathrm{II}', \mathrm{III}' \,|\, 1 + s, -g)\,(s, 1 - g) \\ \mathrm{III} &= (\mathrm{I}', \mathrm{II}', \mathrm{III}' \,|\, s, 1 - g)^2 \end{aligned}\right\} \qquad \ldots\ldots(15),$$

where

$$s = \frac{S' - S}{S - G}, \quad g = \frac{G' - G}{S - G} \qquad \ldots\ldots(16),$$

so that s and g represent the displacements of the stop image and of the object image respectively as a fraction of the original separation between these images.

The values of I, II, III given by (15) if substituted in the expression for $\mathscr{E}_G$ in the equation

$$\mathscr{E}_{G'} = \mathscr{E}_G + \frac{G' - G}{\kappa}\left(\frac{L}{GG'} - L'\right) \qquad \ldots\ldots(17)$$

reduce this to an identity when L and L' are also expressed in terms of I', II', III'. To obtain the desired relations it is necessary to assume that $\mathscr{E}_G$ and $\mathscr{E}_{G'}$ are expanded as infinite series, and the relations follow on regarding (17) as an identity. The most natural form to assume for the expansion is perhaps

$$\frac{1}{\kappa}\,\Sigma c_{p+q+r}\delta_{p,q,r}\,\frac{(p+q+r)!}{p!\,q!\,r!}\,\mathrm{I}^p\,(-2\mathrm{II})^q\,\mathrm{III}^r \qquad \ldots\ldots(18),$$

where δ is the measure of the aberration described by the three integers p, q, r and c_{p+q+r} is a numerical coefficient which it may be found convenient to apply to all aberrations of the same order. The order of the aberration is $n - 1$ where $n = p + q + r$, and the number of coefficients in the terms of order n, and therefore the number of independent aberrations of order $n - 1$, is $\frac{1}{2}(n + 1)(n + 2)$. Of these one relates to the stop only, viz. the term with $p = n$, $q = r = 0$, and those which relate to the image proper thus number $\frac{1}{2}n(n + 3)$. The latter expression gives the well-known value 5 for the first order aberrations, 9 for the second order, 14 for the third, and so on. The transformation formulae which are to be derived, however, relate of necessity to the full number of aberrations, though the aberration at the centre of the stop may not appear in particular cases.

If an expression such as (18) were taken as the basis of the transformation relations the results would appear of needless complexity. To avoid this the aberrations of any order will be arranged in a number of series chosen so that all the terms which appear in the coefficient of $a_u b^v$ in the sum

$$a_0\,(\mathrm{I} - 2\mathrm{II}b + \mathrm{III}b^2)^n + a_1\,(\mathrm{I} - 2\mathrm{II}b + \mathrm{III}b^2)^{n-2}\,(\mathrm{I}\,\mathrm{III} - \mathrm{II}^2) + \ldots\ldots$$
$$+ a_r\,(\mathrm{I} - 2\mathrm{II}b + \mathrm{III}b^2)^{n-2r}\,(\mathrm{I}\,\mathrm{III} - \mathrm{II}^2)^r + \ldots\ldots$$

may be grouped together and considered as an independent aberration. If the power to which $(\mathrm{I}\,\mathrm{III} - \mathrm{II}^2)$ appears is taken as the number of the series, the number

of independent aberrations in each group is shown in the following table, from which it is clear that the rearrangement provides the correct number of independent constants to specify the total number of aberrations which in general are present.

Order of aberration	Total number of aberrations	Number of aberrations of series				
		0	1	2	3	4
1	6	5	1	0	0	0
2	10	7	3	0	0	0
3	15	9	5	1	0	0
4	21	11	7	3	0	0
5	28	13	9	5	1	0
6	36	15	11	7	3	0
$n-1$	$\frac{1}{2}(n+1)(n+2)$	$2n+1$	$2n-3$	$2n-7$	$2n-11$	$2n-15$

Since all the series with the exception of series 0 contain $\mathrm{I\,III} - \mathrm{II}^2$ as a factor, and as for freedom from aberrations every term involving II or III must vanish, in a corrected lens each coefficient of every series but series 0 must vanish. Moreover, when the series are transformed by changes in S and G, the relations which exist between new and old aberrations necessarily connect only aberrations of the same series. For from (5) and (14)

$$(\mathrm{I\,III} - \mathrm{II}^2)(S-G)^2 = (1)(3) - (2)^2 = (\mathrm{I'III'} - \mathrm{II'}^2)(S'-G')^2 = (MN' - M'N)^2 \quad \ldots\ldots(19),$$

which shows that the power of $\mathrm{I'\,III'} - \mathrm{II'}^2$ after transformation is equal to the power of $\mathrm{I\,III} - \mathrm{II}^2$ before transformation. A special consequence of great importance is that apart from the terms of series 0 the conditions for freedom from aberrations do not change with the position of the object or stop, but require the corresponding terms of $\mathscr{E}$ to vanish. On the other hand, if the terms do not vanish the aberrations present will as a rule vary with S and G.

Since for any ray in an axial plane

$$\frac{M'}{M} = \frac{N'}{N},$$

(19) shows that it is only the terms of series 0 which affect the paths of rays in an axial plane. Thus tracing rays in this plane by the usual methods will afford no information about any of the conditions for correction which are independent of S and G. The most advantageous methods of tracing rays in an axial plane will however give all series 1 terms in addition to series 0.

These conclusions may be illustrated by the well-known case of the first order aberrations. Here five of the conditions require adjustment according to the positions of the object and stop, and the state of correction as regards these may be determined by considering rays in an axial plane. There is only one further condition to be satisfied to remove all first order defects. This is the Petzval con-

dition, which, as is well known, is the condition that rays not in an axial plane should conform to the laws which hold for rays in that plane, and which is also quite independent of the position of the stop or of the image. This example also illustrates another property of the constants of higher orders which is related to the facts that have been mentioned. Just as the Petzval condition is particularly simple in form in comparison with the conditions of the lower series, so in the higher order aberrations, as the number of the series increases, the expressions for the evaluation of the coefficients in terms of the constructional data of the system become of greater simplicity.

Assuming that the form given is justified, the terms of order n in $\mathscr{E}_G$ may be written, apart from an arbitrary common factor which need only be considered in connection with the terms arising from L and L' in the transformation expressions,

$$(D_0^n, D_1^n, D_2^n \ldots\ldots D_{2n}^n \text{⨝} \text{I}, -2\text{II}, \text{III})^n$$
$$+ (\text{I III} - \text{II}^2)(S-G)^2 (D_2^1, D_3^1, \ldots\ldots D_{2n-2}^1 \text{⨝} \text{I}, -2\text{II}, \text{III})^{n-2}$$
$$+ (\text{I III} - \text{II}^2)^2 (S-G)^4 (D_4^2, D_5^2, \ldots\ldots D_{2n-4}^2 \text{⨝} \text{I}, -2\text{II}, \text{III})^{n-4}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$+ (\text{I III} - \text{II}^2)^w (S-G)^{2w} (D_{2w}^w, D_{2w+1}^w, \ldots\ldots D_{2n-2w}^w \text{⨝} \text{I}, -2\text{II}, \text{III})^{n-2w} \ldots\ldots (20),$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

where the D's are applied in turn to the coefficients of the successive powers of b in the expansion of the proper power of $(\text{I} - 2\text{II}b + \text{III}b^2)$. It will be noted that if the δ's are expressed in terms of the D's, all the D's which appear in connection with any given δ will have the same suffix.

From equations (15)

$$(\text{I}, \text{II}, \text{III} \text{⨝} 1 - b)^2 = (\text{I}', \text{II}', \text{III}' \text{⨝} 1 + s - sb, -g - (1-g)b)^2,$$

and therefore from (17), (19) and (20) if $w \gtrless 0$

$$D_{2w+v}^w = (D_{2w}^w, D_{2w+1}^w, \ldots D_{2n-2w}^w \text{⨝} 1 + s, -s)^{2n-4w-v} (g, 1-g)^v \ldots\ldots (21),$$

where the D with the accent on the left is the aberration coefficient for the new values of S and G, those on the right being of course the corresponding coefficients for the old values.

This result requires modification for the special case $w = 0$ owing to the presence of the terms in L and L' in equation (17). The additional terms of order n are evidently

$$-\frac{G'-G}{\kappa} \frac{(2n-2)!}{2^{2n-1}(n-1)!\,n!} \left\{ \frac{(S'^2\text{I}' - 2S'G'\text{II}' + G'^2\text{III}')^n}{GG'} - (\text{I}' - 2\text{II}' + \text{III}')^n \right\}.$$

It is convenient to adopt for the value of the constant c the numerical coefficient occurring in this term. Thus

$$c_n = -\frac{(2n-2)!}{2^{2n-1}(n-1)!\,n!} \qquad \ldots\ldots (22).$$

The quantities expressing the aberrations are also preferably pure numbers, so that the factor $1/\kappa$ which introduces the unit of length must also be excluded. The relations for the aberrations of the zero series may then be written

$$D_v^0 = (D_0^0, D_1^0, \ldots D_{2n}^0 \text{⨝} 1 + s, -s)^{2n-v} (g, 1-g)^v + (G'-G)(S'^{2n-v} G'^{v-1} G^{-1} - 1)$$
$$\ldots\ldots (23).$$

It will be observed that the aberration relating to the stop in this notation bears the suffix 0, and the successive aberrations as the suffixes increase proceed in the direction opposite to that adopted by Seidel for first order aberrations. The order adopted is obviously quite arbitrary.

The notation that has been used in this discussion requires a slight alteration to avoid ambiguity in dealing with general aberration problems. To this end the notation employed is to write $D_{n,w,v}$ for the quantity that has hitherto been denoted by D_v^w. The first suffix n gives the order of the terms, so that the aberration is of order $n-1$; the second indicates the series to which the aberration belongs; and the third determines the particular member of that series which is required. The earlier notation may be advantageously employed when the value of n is not in doubt.

The arrangement of aberrations in series.

In obtaining relations (21) and (23) it has been assumed that (18) and (20) are equivalent expansions for $\mathscr{E}_0$. Justification for this assumption beyond the equality in the number of coefficients appears desirable, and the method followed is to express both in terms of a corresponding number of arbitrary coefficients. The proof of these relations is tedious rather than difficult, and is omitted here*. The result is given by the two equations

$$\delta_{n-m+p,\,m-2p,\,p} = \sigma_{n,0,m} + p\sigma_{n,1,m}(S-G)^2 + \ldots + \frac{p!}{u!\,(p-u)!}\sigma_{n,u,m}(S-G)^{2u} + \ldots \qquad \ldots\ldots(24)$$

$$\text{and } D_{n,t,m} = \frac{2^t n!}{t!\,(n-2t)!}\,\frac{1}{\prod_{v=1}^{t}(2n-2t-2v+3)}\left[\sigma_{n,t,m} + \frac{(m-2t)(m-2t-1)}{2\,.\,1!\,(2n-4t-1)}\sigma_{n,t+1,m}(S-G)^2 + \ldots + \frac{(m-2t)!\,\sigma_{n,t+u,m}(S-G)^{2u}}{2^u\,.\,u!\,(m-2t-2u)!\,\prod_{v=1}^{u}(2n-4t-2v+1)} + \ldots\right] \qquad \ldots\ldots(25),$$

where the σ's are arbitrary. It will be observed that in (25) the factor outside the bracket is common to all aberrations of the same series. In applying these formulae it is to be borne in mind that in (24) it is assumed that $m \not< n$. When this condition is not fulfilled it is obvious from symmetry that p should be replaced on the right side of the equation by $n-m+p$. Equations (24) and (25) suffice to establish the general connection between the alternative methods of specifying the aberrations. It is evident that if desired the δ's may be expressed directly in terms of the D's by eliminating the σ's. This elimination is most easily effected by starting

* A copy of a demonstration of the results quoted has been filed by the Hon. Secretary, and can be consulted by arrangement with him.

from the series of highest index number. The converse relation may be written down at once from the reversal of (24)

$$(S-G)^{2l}\,\sigma_{n,t,m} = (\delta_{n-m,m,o},\ \delta_{n-m+1,m-2,1},\ \dots\ \delta_{n-m+t,m-2t,t}\,\overline{\wedge}\,1,-1)^{t},$$

and enables the D's to be evaluated when all the δ's have arbitrary values assigned to them.

The aberrations in terms of the coefficients of $\mathscr{E}$.

Frequently it is of value to be able to express the coefficients in terms of the corresponding coefficients of the standard function $\mathscr{E}$ in which the principal foci are the reference points; and conversely to express the latter coefficients in terms of the D's. The formulae involved are easily found. For if the form assumed for $\mathscr{E}$ is a function of (1), (2), (3) corresponding to that for $\mathscr{E}_G$ in terms of the D's when I, II, III are the variables, the coefficients for $\mathscr{E}$ being indicated by substituting E for D, the relation

$$(S-G)^{2}\,(\mathrm{I},\mathrm{II},\mathrm{III}\,\overline{\wedge}\,1,-b)^{2} = ((1),(2),(3)\,\overline{\wedge}\,1-b,-G+Sb)^{2},$$

and the converse

$$((1),(2),(3)\,\overline{\wedge}\,1,-b)^{2} = (\mathrm{I},\mathrm{II},\mathrm{III}\,\overline{\wedge}\,S-b,-G+b)^{2},$$

in conjunction with (19) at once yield by (3)

$$E^{w}_{2w+v} = (D^{w}_{2w}, D^{w}_{1w+1}, \dots D^{w}_{2n-2w}\,\overline{\wedge}\,1,-1)^{2n-4w-v}\,(G,-S)^{v}\,(S-G)^{2w-2n}\ \dots\dots(26),$$

and

$$D^{w}_{2w+v} = (E^{w}_{2w}, E^{w}_{2w+1}, \dots E^{w}_{2n-2w}\,\overline{\wedge}\,S,-1)^{2n-4w-v}\,(G,-1)^{v}\ \dots\dots(27),$$

if $w \nless 0$. When $w = 0$ additional terms arise from the L and L' terms. In the earlier formula evidently only E^{0}_{0} and E^{0}_{2n} are affected, and the additional terms on the right are $1/G$ in the case of E^{0}_{0} and G in the case of E^{0}_{2n}. The modified form of (27) is

$$D^{0}_{v} = (E^{0}_{0}, E^{0}_{1}, \dots E^{0}_{2n}\,\overline{\wedge}\,S,-1)^{2n-v}\,(G,-1)^{v} - G - S^{2n-v}\,G^{v-1}\dots\dots(28).$$

The parallel with previous expressions can readily be extended further. For example, if in conformity with (25) the E's are expressed by coefficients $\epsilon_{n,t,m}$ corresponding to the σ's, the coefficients of $\mathscr{E}$ will be of the form corresponding to (24) but with S and G absent, and the σ's of (24) and (25) may be expressed as symmetrical functions of the ϵ's of the same order and series.

Special application.

The formulae that have been found may be used to determine the effect of small changes in a system, such as the variation of a thickness or separation, or the alteration of a radius, the work involved being much less than if rays are traced through the modified system. The only requirement is that the aberrations of the original system should be known. Suppose, for example, it is desired to find the effect of changing the curvature of the first radius of a system by a specified amount R, which is assumed to be small. As a result of this change the image will be slightly moved by an amount proportional to R, while the position of the stop will be unaffected. Thus regarding the new system as a combination of a thin meniscus

lens in which the curvatures differ by a small amount R together with the original system, the change in spherical aberration is a known multiple (depending on G) of the aberration arising from the meniscus lens plus the change due to giving g a small value proportional to R, s remaining zero. The effect under the latter heading depends only on the original spherical aberration and the coma, equation (23) giving on putting $v = 2n$

$$\begin{aligned}\text{new spherical aberration coefficient}\\ &= (1 - 2ng) \times \text{old spherical aberration coefficient}\\ &\quad + 2ng \times \text{old coma coefficient}\\ &\quad + g\,(S - G)\,(G^{2n-2} - 1),\end{aligned}$$

higher powers of g than the first being neglected.

If an intermediate surface is to be changed the equations are applied to the following part of the system, the coefficients of which are known if a suitable method of calculation of the original system has been followed. Numerical results on this basis can be reached with less labour than by the system adopted by Mr Wilfred Taylor in his recent paper read before this Society*.

Many of the special cases of these formulae are of much importance to the designer. Apart from previous references by the present author in dealing with first order aberrations, the only instance where they appear to have been used is in a paper by Conrady†, who limits himself to the special case $g = 0$, and discusses only first order aberrations. Two results which are physically obvious may be deduced from (21) and (23) by inspection. The first is that if the object is not moved and is free from aberrations except for the first member of each series (D^{w}_{2w}), no aberration of a different type can be introduced by moving the stop. The second is the complementary case when the stop is fixed and the object moved, and only the last aberration of each series is present (D^{w}_{2n-2w}). The same aberration is the only one present in this case also after the object is moved, with the important exception of the aberrations of series 0, which are subject to the influence of the special additional terms involving the change in the position of the object.

* *Trans. Opt. Soc.* 23 (1921–22), 241.

† "The Five Aberrations of Lens-Systems," *Monthly Notices R.A.S.*, 80 (1919), 78.

THE CLASSIFICATION OF OPTICAL INSTRUMENTS

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ABSTRACT. Exception is taken to the classification of optical instruments by the signs of their powers, and an alternative division comprising five classes is proposed, based upon the separation of the four Gaussian constants into two groups according to their signs. This classification cannot be modified by the addition to the system of inverting prisms and the like, and the properties usually associated with the sign of the lens in reality depend upon its class according to the new system. Each class may have systems of positive or of negative power.

The elementary theory of optical instruments which we owe to Gauss has the great merit that attention may be given wholly to the events which take place in the object space and the image space, the method by which the rays are altered in direction and position, and even the position of the instrument which causes these changes, being unimportant. This treatment is highly advantageous when the purpose is to consider the general correlation between incident and emergent rays, but in the practical applications of the theory to the construction and use of real instruments the position of the instrument is of the greatest importance. This would be rendered more apparent if it became customary at a later stage of the theory to consider the position of the instrument in relation to the cardinal points; such a development leads naturally to a classification of instruments in which there are five groups, important special cases such as telescopes being regarded as borderline cases or as members of two groups. The division proposed is more fundamental than the usual separation of lens systems into positive and negative combinations, a distinction which is not generally correct as regards many of the properties usually associated with them.

The properties of a system, including the positions of the extreme surfaces relative to the cardinal points, are given by the four Gaussian constants A, B, C, D*, which satisfy the identity $BC - AD = 1$. The ordinary division depends wholly on the sign of A, the power of the system. The essential theoretical distinction between positive and negative lenses is indicated in Figs. 1 and 2 by means of three incident parallel rays identified by the letters P, Q, R: the position of the instrument is immaterial and both object space and image space are supposed to extend to infinity in every direction. In the classification now proposed the basis is the grouping of the constants according to agreements or disagreements of sign. Let

* These constants A, B, C, D are respectively $\kappa_{1,n}$, $\frac{\partial \kappa_{1,n}}{\partial \kappa_1}$, $\frac{\partial \kappa_{1,n}}{\partial \kappa_n}$, $\frac{\partial^2 \kappa_{1,n}}{\partial \kappa_1 \partial \kappa_n}$ in the notation used in calculations.

the constants of one sign be included in one bracket, and those of another sign in another bracket. There are then six possible groups

$$[ABCD],\ [ABC]\,[D],\ [A]\,[BCD],\ [AB]\,[CD],\ [AC]\,[BD],\ [AD]\,[BC],$$

it being of no consequence which bracket of a pair corresponds to a positive and which to a negative sign. An algebraically exhaustive list would include the groups $[ABD]\,[C]$ and $[ACD]\,[B]$, but these must be excluded since they require $BC - AD$ to be negative. The actual number of classes is one less than the number of these groups, for if a system is considered in the reverse direction B and C are interchanged. It follows that the two groups $[AB]\,[CD]$ and $[AC]\,[BD]$ correspond to a single class of instrument.

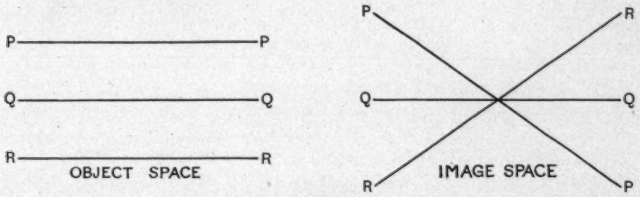

Fig. 1. Positive Lens.

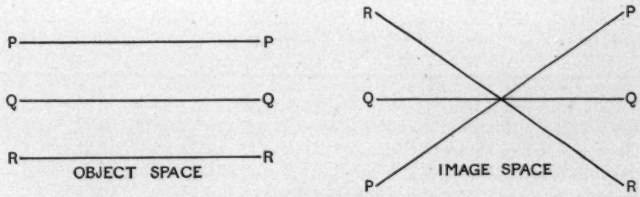

Fig. 2. Negative Lens.

The positions of pairs of conjugate points on the axis are given in terms of A, B, C, D by the relation

$$Axx' - Bx' - Cx + D = 0 \qquad \ldots\ldots(1),$$

where x is the distance from the first surface to the object point, and x' is the distance from the last surface to the image point, both considered positive when measured away from the system into the surrounding medium. It is at once evident from this equation that the positions of conjugate points depend upon the ratios of these constants, and that their relative signs rather than their absolute signs are important. The particular property affected by a change of all the signs without a change of magnitude of these four quantities is the inversion of the image of an object occupying a particular position, but in modern instruments

the designer need pay little attention to this feature of a system as he has several devices which he may adopt to modify this result without altering the values of the four constants. The properties represented by the ratios of the constants on the other hand are fundamental and cannot be influenced by such devices as the introduction of reflecting surfaces.

From the equation connecting x and x' it is easy to see what properties are associated with each of the groups that have been described. Suppose that F (Figs. 3 to 8) is the point at which incident parallel rays are brought to a focus, that U is the image of the vertex of the first surface in the system, and that V is the vertex of the last surface. Then F, U, V are three points on the axis in the image space of which V is necessarily real while F and U are only real if they are encountered by a wave which has traversed the system after it has passed through V. There are six cases to consider. For U may be encountered by the wave before it reaches F or after it has passed through F, and V in either case may precede both,

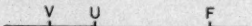

Fig. 3. [ABCD]

Fig. 6. [AC] [BD]

Fig. 4. [ABC] [D]

Fig. 7. [A] [BCD]

Fig. 5. [AB] [CD]

Fig. 8. [AD] [BC]

fall between or follow both U and F. If the points are placed in the order in which they are met by the wave, the following correspondences may be derived at once from equation (1):

$$VUF = [ABCD], \qquad VFU = [AC]\,[BD],$$
$$UVF = [ABC]\,[D], \qquad FVU = [A]\,[BCD],$$
$$UFV = [AB]\,[CD], \qquad FUV = [AD]\,[BC].$$

The general properties of members of any class may be inferred either from equation (1) or from the order of the letters F, U, V if it is remembered that the image of the real part of the object space axis extends from F in the positive direction (through infinity if necessary) to U. Either method shows that in every group with the exception of the last, $[AD]\,[BC]$, there is a range of the axis in which a real image of a real object may be formed. The portion of the image axis which corresponds to the real object space axis is indicated by a thicker line in Figs. 3 to 8.

Among the special cases it will be noted that thin lenses, which have $B = C = 1$

and $D = 0$, are members, if the power is positive, of groups $[ABCD]$ and $[ABC]$ $[D]$; if the power is negative they belong to groups $[A]$ $[BCD]$ and $[AD]$ $[BC]$. When the thicknesses are small without being exactly zero the groups are those mentioned second in each case. It is because the negative lenses ordinarily used belong to the class $[AD]$ $[BC]$, and not because the power is negative, that all the images they form are virtual. It is not difficult to construct positive combinations which have this property.

Other special classes comprise the telescopes. For instance, prismatic binoculars, gun-sighting telescopes, and astronomical telescopes belong to either $[ABCD]$ or $[A]$ $[BCD]$. On the other hand, Galilean binoculars belong to $[ABC]$ $[D]$ or $[AD]$ $[BC]$, and their undesirable properties are wholly attributable to the class to which they belong and have no connection with the erectness of the images they yield.

The adoption of this classification will lead naturally from the Gaussian treatment of rays to the consideration of entrance and exit pupils, an important branch of geometrical optics which has not until lately, in this country at all events, received a proper amount of attention. It would be convenient to have short descriptive titles for each group, but the writer wished to have the opportunity of hearing the views of other workers before making any proposals of this kind.

DISCUSSION

Mr A. Whitwell asked whether it would not be necessary to work out a system from its constructional data in order to see to which class it belonged.

Commander T. Y. Baker (communicated):

Mr Smith's proposal for a classification of optical systems in accordance with the signs of the four Gaussian constants is an exceedingly good one. So far we have had no such classification except as regards the optical power, i.e. the coefficient A. As the author of the paper points out, the physical peculiarities of optical systems that are of vital importance to the designer are dependent quite as much or even more upon the signs and magnitudes of B, C, and D, and a classification on the lines that Mr Smith suggests would, I think, be of more than purely theoretical interest.

I doubt very much, however, whether the nomenclature of classification that Mr Smith advocates will be practically satisfactory because such a symbol as $[AB]$ $[CD]$ conveys no meaning at all apart from the equation

$$Axx' - Bx' - Cx + D = 0$$

and this equation must always be borne in mind together with the convention of the signs of x and x'. I suggest that a more practical method is to classify optical systems according as the principal foci and exit pupils are real or virtual.

Mr Smith shows in Figs. 3 to 8 the positions of the exit pupil U and back focus F in each of his six classes. Now the distance $VU = D/B$ and the distance $VF = C/A$ so that the exit pupil and the back focus are real only when D and B, and A and C are of the same sign respectively. In just the same way if light passes

through the system in the reversed direction we get a real front focus and a real "reversed exit pupil" only if B and A, and D and C, respectively, are of the same sign. Hence Mr Smith's classification can equally well be expressed in terms of the reality or otherwise of the two principal foci and the two exit pupils; as thus:

	$[ABCD]$	$[AD]\,[BC]$	$[AC]\,[BD]$	$[AB]\,[CD]$	$[ABC]\,[D]$	$[A]\,[BCD]$
Back focus	real	virtual	real	virtual	real	virtual
Exit pupil	real	virtual	real	virtual	virtual	real
Front focus	real	virtual	virtual	real	real	virtual
Reversed exit pupil	real	virtual	virtual	real	virtual	real

It is clear from a glance at these groups that consideration of the reality or otherwise is unnecessary for more than three of the four points as in no group could there be three real images and one virtual or three virtual and one real. The classification is consequently known completely by the reality or otherwise of three of these four points and I think without question the two foci and the exit pupil are the three that are the most practical to use. There are two alternatives, namely real or virtual, for each of three positions so that in all there are eight possibilities but two of them are such that the equation $BC - AD = 1$ could not be satisfied. These two arrangements are

(1) Back focus real, front focus virtual, exit pupil virtual.

(2) Back focus virtual, front focus real, exit pupil real.

Neither of these arrangements is possible in any optical system.

The author, in reply to Mr Whitwell, said that in the case of a concrete instrument the class would be determined by seeing if the images of a distant object were real or virtual for both light directions, and also by making the same test for the image of the first surface in the complete instrument.

The classification exhibited in Commander Baker's table agrees with that suggested in the paper, and is based upon identical principles, with the one exception that Commander Baker proposes to use the exit pupils instead of the images of the extreme surfaces in the remainder of the system. As the exit pupil depends upon the relative sizes of various diaphragms, this alteration seems undesirable, if indeed it does not render the proposed basis impracticable for classifying instruments. Apart from this change the table exhibits the properties of the classes very clearly.

The notation used in the paper is sufficiently concise and convenient to serve during a preliminary discussion of the present proposals. If they are adopted the construction of convenient names and a suitable notation will doubtless require fuller consideration. The difficulty anticipated in remembering the equation and the sign conventions does not in reality arise, for there is a generally accepted convention which governs the sign of the power A, and the signs of the other quantities follow automatically since they may be derived by simple differentiation. It suffices then to note that when A, B, C, D are all positive the critical images are all real. A change in the sign conventions for x and x' will alter the signs of terms in the equation, but will not affect the signs of the four Gaussian constants which appear as coefficients in the equation or the classification based on them.

NOTE ON ACHROMATISM WITH ONE GLASS

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MS. received, 8th May, 1922. Read, 8th June, 1922.

ABSTRACT. Systems composed of thin lenses of the same kind of glass and achromatised by selecting suitable positions for the components are members of the class $[AD]\,[BC]$, so that if the object is real the image is virtual. Apochromatic systems constructed from normal achromatic lenses belong to the same class. The outstanding aberrations for systems constructed of a single glass but belonging to other classes are of considerable magnitude.

Optical systems in which some degree of colour correction is achieved by the use of well separated single lenses of the same glass are almost as familiar as those in which the correction is attained by the juxtaposition of two or more glasses having different dispersive properties. The colour correction ordinarily attained by separation is less complete than with the alternative method, though systems fully corrected for an object in a given position have been described*. It does not, however, appear to be recorded that systems in which achromatism is secured by separation are of the class $[AD]\,[BC]$† in which an object and its image cannot be simultaneously real, though the fact that such lenses are used almost exclusively in subjective instruments would indicate that they had been found chiefly suitable where the image is virtual. The ordinary expression of the conditions for the elimination of chromatic aberrations tends to conceal this result, and it was only in consequence of their failure to secure the desired position of the image in a special system that the present authors were led to investigate the cause of the difficulty and thus to discover the general result which has been stated.

The case of four lenses.

In simple cases it may be shown directly from the form of the expressions for A, B, C, D in terms of the powers and separations of the component lenses that when the system is completely free from colour it belongs to the group $[AD]\,[BC]$. As an example, take the simplest construction consistent with the satisfaction of the three colour conditions, where there are four lenses. The expressions for three of the constants may be written

$$Bt_3 = k_2 - D(1 - t_3\kappa_4),$$
$$Ct_1 = k_3 - D(1 - t_1\kappa_1),$$
$$Dt_2 = t_1t_3 - k_2k_3,$$

* See for instance Sampson "On a Cassegrain Reflector with Corrected Field," *Phil. Trans.* A, vol. 213, p. 27.
† "The Classification of Optical Instruments," *Trans. Opt. Soc.* 23 (1921–22), 323.

where t_1, t_2, t_3 are the three separations, $\kappa_1, \kappa_2, \kappa_3, \kappa_4$ the powers of the four lenses, and

$$k_2 = t_1 t_2 \kappa_2 - t_1 - t_2, \quad k_3 = t_2 t_3 \kappa_3 - t_2 - t_3.$$

As B, C, D are not to vary with the colour, and as each lens is made of the same glass

$$t_1 t_2 \kappa_2 + D t_3 \kappa_4 = 0,$$
$$t_2 t_3 \kappa_3 + D t_1 \kappa_1 = 0,$$
$$t_1 \kappa_2 k_3 + t_3 \kappa_3 k_2 = 0.$$

These conditions in combination with the original equations require

$$B t_3 + D + t_1 + t_2 = C t_1 + D + t_2 + t_3 = 0,$$

$$k_2 = -\frac{t_1 + t_2}{1 - \theta}, \quad k_3 = -\frac{t_2 + t_3}{1 + \theta},$$

$$D t_2 = -\frac{t_1 t_3 \theta^2 + t_2 (t_1 + t_2 + t_3)}{1 - \theta^2},$$

$$(D + t_1 + t_2 + t_3) t_2 = -\frac{(t_1 + t_2)(t_2 + t_3)\theta^2}{1 - \theta^2},$$

where θ is arbitrary. Now all the t's are necessarily positive, and it appears that D must have the same sign as $D + t_1 + t_2 + t_3$; therefore B and C are of like sign and differ in sign from D. Also

$$A = \frac{BC - 1}{D} = \frac{(D + t_2)(D + t_1 + t_2 + t_3)}{D t_1 t_3}$$

and thus A agrees in sign with D. It has thus been proved that the system belongs to the class $[AD]\,[BC]$. It may be noted that the power may be given whatever sign is desired by selecting for θ^2 values greater or less than unity.

The general conditions for achromatism.

A proof similar to the above for a system composed of an indefinite number of elements would be cumbersome and in addition open to the objection that it does not show whether it is one particular condition which limits the class of the instrument or whether this arises from the combination of the several conditions which the system is made to satisfy. The proof given below is accordingly based upon the general expressions for chromatic aberrations. The equations from which these are derived show that the conclusions will still hold when the rays considered are not paraxial.

Consider a ray traversing any system symmetrical about the x axis, and meeting a refracting surface of curvature R_p at the point x_p, y_p, z_p. If after refraction through this surface the ray lies in the direction denoted by the direction cosines L_p, M_p, N_p, the refractive index being μ_p, the y and z refraction equations are of the form

$$y_p K_p + \mu_p M_p - \mu_{p-1} M_{p-1} = 0 \qquad \ldots\ldots(1),$$

and the geometrical relations of the form

$$y_{p+1} - y_p - t_p M_p = 0 \qquad \text{.....(2),}$$

where $$K_p = R_p (\mu_p \cos \phi_p' - \mu_{p-1} \cos \phi_p),$$

ϕ_p and ϕ_p' being the angles of incidence and emergence at the surface considered. If accented letters relate to any other chain of quantities of the same type

$$\begin{aligned} 0 &= \Sigma y_p (y_p' K_p' + \mu_p' M_p' - \mu'_{p-1} M'_{p-1}) - \Sigma \mu_p M_p (y'_{p+1} - y_p' - t_p' M_p') \\ &\quad - \Sigma y_p' (y_p K_p + \mu_p M_p - \mu_{p-1} M_{p-1}) + \Sigma \mu_p' M_p' (y_{p+1} - y_p - t_p M_p) \\ &= \Sigma y_p y_p' (K_p' - K_p) + \Sigma \mu_p \mu_p' M_p M_p' \left(\frac{t_p'}{\mu_p'} - \frac{t_p}{\mu_p} \right) \\ &\quad - \mu_0' M_0' y_0 + \mu_0 M_0 y_0' + \mu_n' M_n' y_{n+1} - \mu_n M_n y'_{n+1} \qquad \text{.....(3),} \end{aligned}$$

where the first summation covers all refractions and the second all elements of the path between the arbitrary end points whose coordinates appear in the last line of the equation. If all the y's of this line are made zero it follows that

$$- \Sigma \mu_p \mu_p' M_p M_p' \left(\frac{t_p'}{\mu_p'} - \frac{t_p}{\mu_p} \right) = \Sigma y_p y_p' (K_p' - K_p) \qquad \text{.....(4)}$$

identically. Suppose now that both sets of equations relate to paraxial rays through the same optical system in which each lens is of the same glass. In place of K_p the paraxial power κ_p must be substituted, and if the accented quantities relate to rays of another colour K_p' becomes $\kappa_p \left(1 + \frac{1}{\nu}\right)$ where ν is a dispersive constant of the glass depending upon the two colours considered. On the left of (4) all the t's and t''s, with the exception of the lengths in the external medium, are necessarily equal, and (4) takes the form, assuming that the system is surrounded by air,

$$- M_0 M_0' \delta x - M_n M_n' \delta x' + \Sigma t_p M_p M_p' \delta \mu_p = \frac{1}{\nu} \Sigma y_p y_p' \kappa_p \qquad \text{.....(5).}$$

If all the lenses are thin the sum on the left is zero, and in the lenses ordinarily used in real instruments the contributions due to their thicknesses are unimportant in comparison with other terms. It is therefore permissible in nearly all cases to regard all the lenses as thin and to write (5) in the form

$$- M_0 M_0' \delta x - M_n M_n' \delta x' = \frac{1}{\nu} \Sigma y_p y_p' \kappa_p \qquad \text{.....(6).}$$

A further simplification which is an essential step in all discussions of this kind is to note that each sum in (4) contains a small term depending on the colour difference, and therefore, as the expressions under consideration only concern first order corrections, the quantities outside these differences may all be identified with rays of one colour. Thus in (6) M_0' and M_n' with y_p' may be given values calculated for a ray of the same colour as M_0, M_n, y_p. In particular the two rays may be considered to be the same ray, and therefore

$$- M_0^2 \delta x - M_n^2 \delta x' = \frac{1}{\nu} \Sigma y_p^2 \kappa_p \qquad \text{.....(7).}$$

This is an expression from which differences of focussing position can be calculated by substituting zero for δx. It appears that the sum on the right can be made to vanish by selecting a system from a number of positive and of negative lenses, in which case the images of two colours will occupy the same plane normal to the axis. The condition therefore for freedom from chromatic aberration of position is

$$\Sigma y_p^2 \kappa_p = 0 \qquad \ldots\ldots(8)$$

where y_p is the distance from the axis at which the thin lens of power κ_p is intersected by a ray from the axial point of the object. If this condition is satisfied for three distinct object points the system will be completely corrected for colour.

The corrected image is virtual.

It will now be shown that if equation (8) is satisfied the image of a real object is necessarily virtual. For equations (1) and (2) are still satisfied if $-t, -y, -K$ are substituted for t, y, K, and therefore from (3)

$$0 = -\Sigma y_p y_p' (K_p' + K_p) + \Sigma \mu_p \mu_p' M_p M_p' \left(\frac{t_p'}{\mu_p'} + \frac{t_p}{\mu_p}\right) + \mu_0' M_0' y_0 + \mu_0 M_0 y_0' - \mu_n' M_n' y_{n+1} - \mu_n M_n y'_{n+1} \qquad \ldots\ldots(9).$$

Assuming that the rays considered are paraxial of the same colour and that the y coordinates of the end points are zero, the relation becomes

$$\Sigma y_p y_p' \kappa_p = \Sigma \mu_p M_p M_p' t_p + (x + \tfrac{1}{2}\delta x) M_0 M_0' + (x' + \tfrac{1}{2}\delta x') M_n M_n' \qquad \ldots\ldots(10).$$

When the two rays become identical

$$\Sigma y_p^2 \kappa_p = \Sigma \mu_p M_p^2 t_p + M_0^2 x + M_n^2 x' \qquad \ldots\ldots(11),$$

$\delta x'$ being negligible in comparison with the other quantities. Every term in the sum on the right is necessarily positive, and therefore if the sum on the left vanishes, that is if the system is corrected for colour for this object position, either x or x' or both must be negative; in other words, either the object or image or both of them are virtual. In particular, if the system is completely corrected for colour there can be no positive x associated with a positive x', and the system belongs to the class $[AD]\,[BC]$.

Apochromatic Systems.

This result may be applied to systems built of normal achromatic thin lenses. For such lenses all have similar errors in the secondary spectrum, and it follows that a combination of achromatic lenses can only yield an apochromatic image if this is virtual.

The evaluation of outstanding defects.

The equations which have been obtained may be readily employed to evaluate the defects unavoidably present in a system when the image is real. Equations (7) and (11) give for thin lenses in air

$$-M_0^2 \delta x - M_n^2 \delta x' = \frac{1}{\nu}[M_0^2 x + M_n^2 x' + \Sigma M_p^2 t_p],$$

and this may be expressed in terms of the transverse magnification m and the powers reckoned from either end of the system by means of the relations

$$mM_n = M_0, \quad M_p\kappa_{1,n} = M_0\kappa_{p+1,n} + M_n\kappa_{1,p}.$$

The result is

$$-\delta x' - m^2\delta x = \frac{1}{\nu}\left[x' + m^2x + \frac{1}{\kappa^2_{1,n}}\Sigma t_p\,(\kappa_{1,p} + m\kappa_{p+1,n})^2\right] \quad \ldots\ldots(12).$$

Thus for an object at infinity, $m^2x = 0$, and

$$-\delta x' = \frac{1}{\nu}\left[x' + \frac{1}{\kappa^2_{1,n}}\Sigma t_p\kappa^2_{1,p}\right].$$

Evidently in any case in which x' has a positive value comparable with that which would be secured if the whole system were thin the chromatic aberration is worse than if a thin lens of the same glass were utilised. As a rule the errors in the two cases will usually be of the same order of magnitude with real images, and systems of this kind are therefore unsuitable for use where a real image is required. Numerical investigation in a number of systems corrected for other image defects when the object is at infinity has shown that the errors are rarely less than twice those of a single lens of the same focal length, and are frequently considerably more.

By substituting for x and x' in (12) their values in terms of the constants A, B, C, D the formulae

$$\left.\begin{aligned}
\nu\delta A &= A - \Sigma t_p\kappa_{1,p}\kappa_{p+1,n} \\
\nu\delta B &= -\Sigma t_p\frac{\partial\kappa_{1,p}}{\partial\kappa_1}\kappa_{p+1,n} \\
\nu\delta C &= -\Sigma t_p\kappa_{1,p}\frac{\partial\kappa_{p+1,n}}{\partial\kappa_n} \\
\nu\delta D &= -D - \Sigma t_p\frac{\partial\kappa_{1,p}}{\partial\kappa_1}\frac{\partial\kappa_{p+1,n}}{\partial\kappa_n}
\end{aligned}\right\} \quad \ldots\ldots(13)$$

are obtained, and the variation of the magnification may be found either from these expressions or directly by substituting $M_0 = M_n' = 0$ in (3) and (9). As a rule equations (13) are replaced by the more symmetrical forms

$$\left.\begin{aligned}
\nu\delta\left(\frac{1}{A}\right) + \frac{1}{A} &= \frac{1}{A^2}\Sigma t_p\kappa_{1,p}\kappa_{p+1,n} \\
\nu\delta\left(\frac{B}{A}\right) + \frac{B}{A} &= -\frac{1}{A^2}\Sigma t_p\kappa^2_{p+1,n} \\
\nu\delta\left(\frac{C}{A}\right) + \frac{C}{A} &= -\frac{1}{A^2}\Sigma t_p\kappa^2_{1,p}
\end{aligned}\right\} \quad \ldots\ldots(14),$$

which are derived from (12) by regarding it as an identity in m. Still other forms are available for the ratios to D by expressing m in terms of x and x'.

AN IMPROVED SUBJECTIVE TEST FOR ASTIGMATISM

By HERBERT S. RYLAND
(Messrs Archer Croft & Co., London)

MS. received, 15th May, 1922. Read, 8th June, 1922.

With the usual test card, consisting of a series of lines, many patients have difficulty in determining which line is most distinct and when; also, if the ametropia is of high degree, say ± 2·00 *D* or more, the test has to proceed to a certain extent before the astigmatism is apparent.

The present test has been designed to obviate these difficulties. It consists of an opaque disc perforated along two diameters at right angles with a series of square apertures (Fig. 1). These apertures and the distances between them subtend angles of 1′ at the usual testing distance. The plate is illuminated by diffused light from the rear.

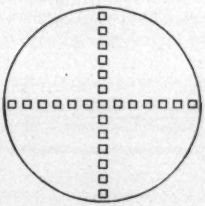

Fig. 1

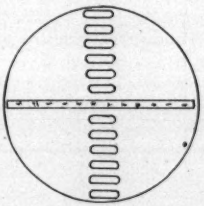

Fig. 2

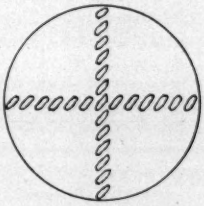

Fig. 3

In use the patient is directed to observe the instrument with one eye covered. The disc is slowly turned until one line of apertures is seen as a continuous line, while that at right angles appears as a series of dashes at right angles to its length (Fig. 2). This will occur if astigmatism is present, and so far the author has been unable to find a patient without sufficient astigmatism to detect easily by this means.

The differentiation between the lines is marked even with high degrees of ametropia and in consequence the refraction is obtained much more readily than by the usual methods. With 0·25 *D* of astigmatism the axis is easily located to ± 1°, since in any position other than with the lines parallel to the principal meridian the lines appear as a series of oblique dashes (Fig. 3).

In practice the instrument is made self-contained. It may be obtained from Messrs Newbold & Co., 72, Gray's Inn Road, W.C.

THE MECHANICAL CONSTRUCTION OF THE MICROSCOPE FROM A HISTORICAL STANDPOINT, WITH SPECIAL REFERENCE TO CERTAIN INSTRUMENTS NOW IN THE SCIENCE MUSEUM, SOUTH KENSINGTON

By Professor ALAN POLLARD, A.M.I.E.E., F.Inst.P.

Read, 27th April, 1922.

The word microscope appears to have been first suggested by Demiscianus, an eminent Greek scholar, who also at the request of Prince Cesi, president of the Lyncean Academy, invented the word telescope. During the early part of the 19th century Dr Goring, who did so much for the development of the instrument, proposed the name engyscope for the compound microscope as distinct from the microscope or single lens. The suggestion of Demiscianus has however happily survived the eroding action of ever flowing time.

The simple lens appears to have had its birth with the ancients. We know that Aristophanes, Seneca, Pliny, who died in 79 A.D., and Ptolemy, the famous astronomer of Alexandria, were well acquainted with the altered appearances of objects seen through spheres of glass or transparent globes of water. But though the Greek and Roman physicians were acquainted with the common defects of the sight, they do not appear to have included in their treatment the use of lenses. Claudius Galen, the most celebrated of ancient medical writers, who was born about 130 A.D., and whose name figures in the early history of stereoscopic vision, makes no mention of lenses as aids to medical art.

The simple biconvex lens, according to Dr Francis Redi, was in use early in the 4th century, but this does not appear to have been generally known, for Record in his work, *The Pathway of Knowledge, containing the first Principles of Geometry, as they may most aptly be applied unto practice both for use of Instruments Geometrical and Astronomical, etc.* London, 1551, Reginald Wolfe, relates that Roger Bacon while at Oxford made a glass which exhibited such curious things that its effect was generally attributed to some diabolical power. Roger Bacon was born in 1214 and died in 1292.

The earliest known illustration of a simple microscope is figured on one of the plates in Descartes' *Dioptrice*, the date of which is 1637. Thus the discovery of the simple lens, which is still shrouded in obscurity, led up to a design of coacting parts, i.e. an instrument, by Descartes which was the first suggested simple microscope. I am not aware that actual models of Descartes' microscope have ever been found. The invention of the compound microscope, according to the researches of Professor Govi which cannot be lightly refuted, is due to the illustrious Galileo, who, it appears, developed the compound microscope from the Dutch telescope prior to 1610, whilst Drebbel in 1621 reduced the Keplerian telescope to a microscope. Thus there are two possible sources from which the compound microscope may have sprung. The

achievement of Galileo was announced by one of his pupils, John Wodderborn, a Scotchman, in an account published in 1610 which reads thus:

I will not now attempt to explain all the perfections of this wonderful *occhiale*; our sense alone is a safe judge of the things which concern it. But what more can I say of it than that by pointing a glass to an object more than a thousand paces off, which does not even seem alive, you immediately recognise it to be Socrates, son of Sophronicus, who is approaching? But time and the daily discoveries of new things will teach us how admirably the glass does its work, for in that alone lies all the beauty of that instrument. I heard a few days back the author himself (Galileo) narrate to the Most Excellent Signor Cremonius various things most desirable to be known, and amongst others in what manner he perfectly distinguishes with his telescope the organs of motion and of the senses in the smaller animals; and especially in a certain insect which has each eye covered by a rather thick membrane, which however, perforated with seven holes, like the visor of a warrior, allows it sight. Here hast thou a new proof that the glass concentrating its rays enlarges the object; but mind what I am about to tell thee, viz. in the other animals of the same size and even smaller, some of which have nevertheless brighter eyes, these appear only double with their eyebrows and the other adjacent parts.

In the Museo di Fisica there are two small microscopes which tradition affirms were constructed by Galileo, and illustrations of which are to be seen on p. 129 of *The Microscope* by Carpenter, 1891. If this is true it is with these instruments that the history of the mechanical construction of the microscope commences. The body is mounted on a rigid tripod of better design than that of many later instruments and the adjustment is performed by screwing or unscrewing the tube in the primitive limb of the stand. This form of adjustment persisted up to the time when Cuff tried to improve the Culpeper model of 1735 by adding a screw for fine adjustment at the top of the nose-piece. Possibly these instruments were not Galileo's occhiale at all but were early forms of an interesting compound microscope by Giuseppe Campani described in 1686, and illustrated on p. 129 of Carpenter's work of 1891. Campani's microscope is similar in mechanical construction to the so-called Galilean instruments.

As already stated, Cornelius Drebbel, a Dutch chemist, according to Huygens and other authors, invented the compound microscope in 1621, whilst Fontana, a Neapolitan, claims the discovery for himself in 1618. Borellus however claims Zacharius Jansen or his father Hans, who were spectacle makers in Holland, as the originators in 1590 and they are said to have presented the first microscope to Charles Albert, Archduke of Austria. "One of their microscopes," says Sir D. Brewster in his *Treatise on Microscopes*, "which they presented to Prince Maurice was in the year 1617 in the possession of Cornelius Drebbel of Alkmaar, who then resided in London as mathematician to King James I, in which place he made microscopes and passed them off as being of his own invention." These instruments were said to be six feet in length and consisted of a tube of gilt copper, one inch in diameter, supported by thin brass pillars, in the shape of dolphins on a base of ebony, which was adapted to hold the object to be examined.

No doubt they were similar to one which Aepinus has described in a letter addressed to the Academy of Sciences of St Petersburg. Whether this is true or not, at the Loan Collection of Scientific Instruments in London in 1876 an old micro-

scope which had been found at Middelburg was shown, which Professor Harting considered might possibly have been made by the Jansens. A copy of this instrument was made by Beck and is now exhibited in the Museum. As it consists merely of iron tubes holding the two lenses and was intended to be used in the hand like a short range telescope, it is not of much mechanical interest except perhaps in the manner in which the lenses were mounted in wooden cells which were held in position by a wire ring sprung in, and in the fact that the diaphragms can be adjusted. A sprung-in wire ring is used to-day in cheap optical instruments.

We come now with a leap, so to speak, as far as mechanical interest is concerned, to a remarkable instrument designed by a remarkable man, i.e. the compound microscope of Robert Hooke which is described in his *Micrographia* published in 1665. This instrument is pictured in the same edition of Carpenter's book, p. 130. Previous to this in 1660 Monconys devised and had made the earliest device of a microscope with a field lens. Hooke employed the field lens of Monconys in his instrument, not to correct the spherical aberration of the eye lens, but merely to increase the size of the field of view. He tells us he removed this field lens when he wanted to examine detail on an object. The large body of Hooke's instrument screws at its lower end into a ring which is connected by means of a ball and socket joint to another ring that may be clamped in any position upon a vertical rod fixed to the base plate. By turning the body the screwed end gives it axial translation and serves as a fine adjustment, while the clamp upon the rod acts as a coarse movement. The stage is movable upon the base plate and is a simple and practical arrangement. A globe and lens with a lamp, all clamped to a separate stand, act as a powerful illuminating device.

It is interesting to note that Hooke was the first to describe the well known method of estimating the magnifying power of a compound microscope by observing the image with one eye and a scale placed in the same plane as the object with the other. Hooke also was the first to make minute globular lenses of high power, which achievement was claimed by Hartsoeker, though the *Micrographia* was published the same year that Hartsoeker was born.

Coeval with Hooke were Eustachio Divini of Rome and Campani of Bologna and in 1672 S. P. Salvetti made microscopes in imitation of those of Divini and Campani, but these are more interesting from the optical point of view.

It was in the year 1672 also that Newton* communicated to Oldenburg his "design of a microscope by reflection, which should have, instead of an object glass, a reflecting piece of metal, and which seemed as capable of improvement as telescopes and perhaps more so, because but one reflective piece of metal is requisite in them." This idea was improved later by Smith, Barker, Amici, Potter, and Goring, who made the reflector a portion of an ellipsoid and fixed a small piece of speculum between it and the focus. By this means the object placed to one side for easy illumination is reflected upon the ellipsoidal reflector. In another letter to Oldenburg dated July 11th, 1672, he makes an important suggestion upon refracting microscopes.

* *Memoirs of the Life, Writings and Discoveries of Sir Isaac Newton*, by Sir David Brewster, vol. 1, p. 242.

Which I do (he says), more willingly, because Mr Hooke hath made such excellent use of that instrument; and I shall be glad to contribute anything to your promotion of these his ingenious endeavours or add to his inventions of that kind. The way is by illuminating the object in a darkened room with light of any convenient colour not too much compounded; for by that means the microscope will with distinctness bear a deeper charge and larger aperture, especially if its construction be such as I may hereafter describe.

He does not appear however to have described this construction.

This important notion was realised by Brewster* by means of sodium light, and it is a pity that Newton's suggestion is not more frequently acted on in the design of objectives to-day for many possible scientific purposes. This seems to be all that Newton contributed to microscopy.

During the next year 1673 Anthony van Leeuwenhoek (1632–1723) began to contribute to the *Philosophical Transactions* his microscopical discoveries which he achieved by his now famous simple microscopes. At his death he bequeathed to the Royal Society a cabinet containing 26 of his microscopes which are now lost. Martin Folkes† vaguely reported upon them; he describes the lens as a very small double convex glass let into a socket between two silver plates riveted together and pierced with a small hole. He says "...being all ground glasses none of them are so small and consequently magnify to so great a degree as some of those drops, frequently used in other microscopes."

In 1740 these microscopes were examined and described to the Royal Society by Baker‡. No figure was published by Baker until 1753 when he issued two outline drawings in his book *Employment for the Microscope*, 1st ed. 1753. These drawings do not give a clear idea of the construction of the instruments.

Professor A. W. Hübrecht, however, brought one of these microscopes, belonging to the Zoological Society of the University of Utrecht, to the Royal Microscopical Society, where careful drawings and models were made. A picture of one of these models, with a bi-convex lens of $\frac{1}{4}''$ focus, is to be found in Carpenter's work, where it is fully described.

Improvement in the design of instruments invariably follows in the wake of scientific discoveries. There can be no doubt that William Harvey, a young Englishman pursuing his studies at Padua under the anatomist Fabricius between 1598 and 1610, powerfully stimulated the improvement of the microscope; these studies enabled him to announce in 1628 in his treatise *De Motu Cordis et Sanguinis* his great generalisation of the circulation of the blood. The circulation in the capillaries was discovered by Marcellus Malpighi of Bologna in 1661 in a letter addressed to Borelli, four years after Harvey's death. Leeuwenhoek in 1673 repeated Malpighi's observations and studied the capillary circulation in a bat's wing, the tail of a tadpole and the tail of a fish.

We find that in 1693 John Marshall produced his "new invention double microscope" for viewing the circulation of the blood. A perfect specimen of Marshall's microscope, to be seen in the Court Collection in the Museum, is shown in Fig. 1. Baker stated in 1742 that it was later improved upon by Culpeper and Scarlett, but

* *Edin. Trans.* vol. 9, p. 433; also *Edin. Journ. of Science* (New Series), No. 1 (July, 1829), 108.
† *Trans. Roy. Soc.* 32 (1723), 446–463. ‡ *Ibid.* 41 (1740), 503–519.

we shall see that this is not really the case. It was a distinct improvement upon all previous models in the fine adjustment which moves the whole body in the stage, which is attached to the limb, so that it is integral with the body when the instrument

Fig. 1

is tilted. It will be noted that the instrument is supplied with a lead coffin for weighting down a fish while observing the capillary circulation in the tail. Arrangements for holding fish were supplied with instruments for many years to come. Previous to Marshall, Bonnani in 1698 describes a compound microscope in use by him. This

instrument is remarkable in that a compound condenser is used for the first time, consisting of two bi-convex lenses. The fine adjustment is that used by Hooke, but a rack and pinion functions for the coarse adjustment.

The compound microscope at this time however was so unwieldy and its optical system so inferior to the single lens that there was no great demand for it for really scientific work, and for a considerable time the single lens held the field.

A system of focussing was employed by Hartsoeker in 1694 for the hand simple microscope which was adopted and improved by Wilson. It was figured and described in the *Philosophical Transactions* in 1702. This instrument was made by all the leading opticians, Edmund Culpeper, John Cuff, Martin, Mann and Ayscough, and Scarlett throughout the 18th century.

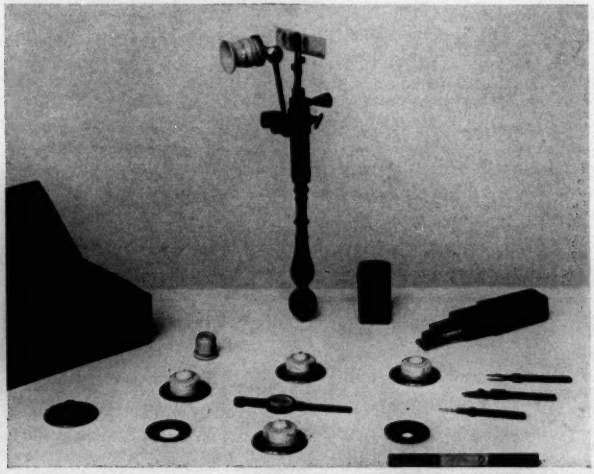

Fig. 2

Between the years 1702–1706 Wilson considerably improved its construction and added an ivory handle, a holder for opaque objects, a true spiral spring to the focussing part, and an ivory object plate, black on one side and white on the other, for light or dark objects respectively; several of Wilson's instruments are to be seen in the Court Collection in the Museum. With this screw barrel microscope Wilson supplied another for opaque objects, made on the compass principle of Musschenbroek and Cuno, and the powers were common to both. In this condition they were sold as a "set of pocket microscopes."

Fig. 2 shows a compass microscope by Culpeper. Wilson also supplied a pedestal

to which either of his microscopes could be attached, and he suggested that the light used might be that reflected from a looking glass. Later, no doubt acting on this suggestion, Culpeper, Scarlett and other 18th century opticians as well as Wilson himself fitted the instrument to either a tripod or scroll stand provided with a mirror, and Fig. 3 shows a fine example of a Wilson scroll microscope made by Cuff, which is taken from the Court Collection.

Occasionally a rack and pinion adjustment was added, and there is a model of 1771 by Martin with such a rack adjustment, and an objective changer, which is a brass slide with six powers, in the Museum. Cuff about 1740 added the Lieberkühn. The objective slide is of interest, for it is the first attempt to provide a ready means

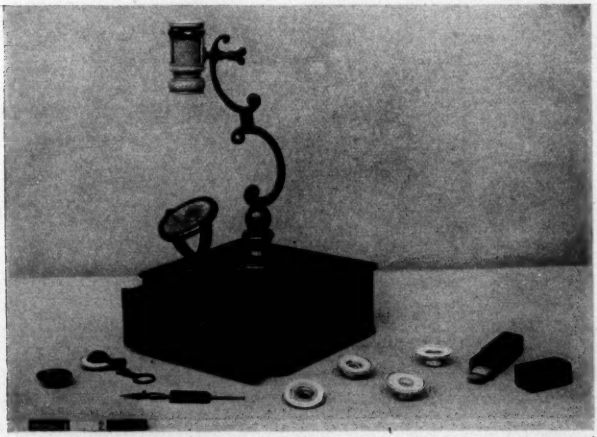

Fig. 3

for rapidly changing the objective. It is due to either Martin or George Adams who designed a microscope also in 1771 with such a changer.

The mirror however did not originate with the later models of the Wilson instrument, for we find that Hertel in 1716 employed not only a mirror below the stage, but also a concave metal mirror above the stage for reflecting light through a condenser on to the object. Moreover Hertel's instrument is of further interest in so far as he hinged the body tube about a joint at the top of the supporting pillar and fixed it in position with a screw bent into the arc of a circle which threaded a nut rotatable in the pillar. It is curious that he did not adopt the improvements of Marshall for he made the tube only inclinable with no mechanical focussing arrangements, but arranged that the stage and mirror should be moved by

complex mechanism hidden in the box-like stand. Thus one maker ignored the improvements of his predecessors and the evolution of design hobbled painfully along.

We next find the introduction by Edmund Culpeper in or a few years before 1735 of a design which was known as the "double reflecting microscope." This microscope is a refracting instrument and its name no doubt was intended to draw attention to the mirror. A very fine example of this microscope is to be seen in the Court Collection labelled E. 7—1908. It was first described by Dr R. Smith in 1738. At that time it was fixed on an octagonal box foot, but in the early instruments made by Culpeper the base is circular and without a drawer for the apparatus. The tripod form of stand mounted in this way was a favourite model for over 100 years and was made during this time by all the leading English opticians with various modifications in design, material and finish. As I have said, Baker stated that this instrument was an improvement on Marshall's large double microscope, which is true as far as clumsiness is concerned, but the two have little in common except the box foot.

The coarse adjustment was obtained by sliding the inner tube to a mark corresponding to the number of the objective used, there is no fine adjustment but a nicer focus could be obtained by unscrewing the objective on the nose-piece. The eye-piece was composed of an eye lens and a field lens. In the earlier models made of wood and cardboard there was no diaphragm, but this was often added in the later construction in brass. There were five objectives or "buttons," each consisting of a double convex lens. No. 1 was the highest power, No. 2 the next and so on, the focal length varying from about $\frac{1}{12}$" to 1".

As I have already said, Cuff tried to improve the Culpeper model by adding a screw for fine adjustment at the top of the nose-piece. Scarlett and Cuff improved it by making the three supporting legs in one piece and thus increased the rigidity. which is clearly shown in the photograph, Fig. 4.

W. H. Harris of London and Hamburg however greatly improved this type of microscope in 1830 by making it entirely of brass and by effecting the focussing by rack and pinion. Being of simple construction it could be made and sold cheaply, and this accounts for the persistence and continual improvement of an obsolete type at a time when many instruments of a very much better design were being produced. An example of this instrument is labelled in the Court Collection as E. 141—1911. The mechanical design of this instrument was retrograde—no use was made of Bonanni's rack and pinion, no use was made of Marshall's fine adjustment, nor was any attempt made to incline the instrument for the comfort of the user. These mechanical improvements did not appear combined in any one instrument until the middle of the 18th century, more than 50 years after they had been separately invented and successfully applied.

In 1738 Dr Lieberkühn of Berlin devised what had been suggested by Descartes nearly a century before, the instrument that has ever since been known by his name. He also devised the solar microscope. Lieberkühn was a histologist of high order, who discovered the tubular glands in the small intestines, which bear his name. He

was also celebrated for his many beautiful injections of the vascular tissues and organs of vertebrates.

In 1742 Martin, an incomparable mechanician of his time, described his "pocket

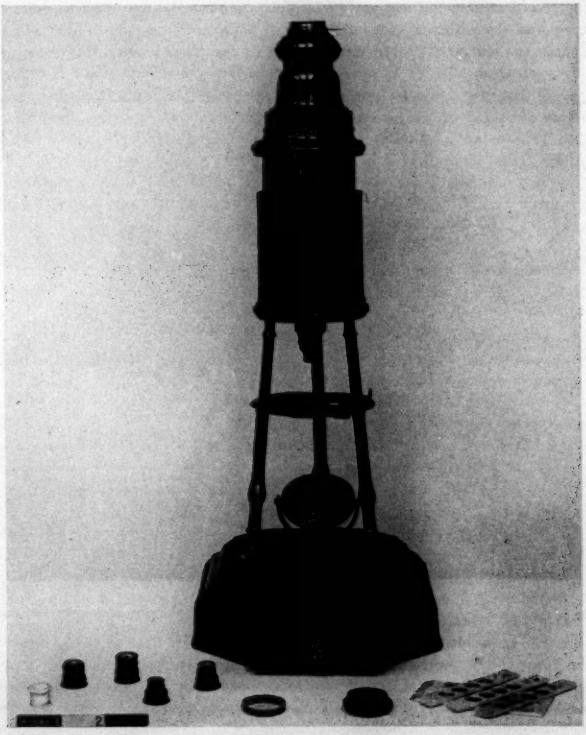

Fig. 4

reflecting microscope" which had a micrometer eyepiece with a screw of a certain number of threads per inch and by which accurate measurements could be taken. It was a dioptric instrument but had a mirror, from which its name is taken, fitted into its cylindrical base, and is probably the original form of the modern "drum"

microscopes. Martin was well qualified to construct micrometric apparatus and we find Quekett describing a large universal microscope by Martin with a mechanical stage having rectangular movements and a fine adjustment. The rectangular movements were actuated by fine threaded screws bearing milled heads which were graduated, and this is the first mechanical stage in the history of the microscope.

At this time besides Benjamin Martin there resided in the metropolis the skilled mechanicians John Cuff and George Adams, and we find that in 1744 Cuff, at the instance of Baker's complaints concerning the Culpeper design, produced an instrument which was an important step in the evolution of the modern microscope. This "double constructed microscope," as he called it, had a fine adjustment similar

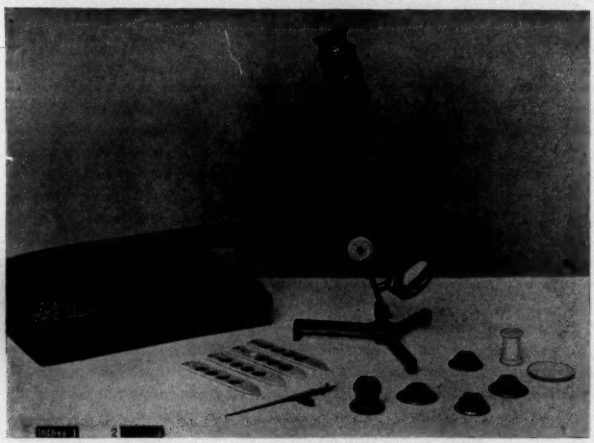

Fig. 5

to Marshall's. The limb was a fixed pillar up and down which the compound body could slide to figured marks corresponding to the objectives used. The stage and mirror were more accessible and the whole instrument was firmer and more rigid.

In 1747 Cuff invented a micrometer for his instrument, which was a rectangular lattice of fine silver wires, $\frac{1}{50}''$ apart, placed in the focus of the eyeglass, so as to divide the field into squares each $\frac{1}{50}'' \times \frac{1}{50}''$. An example of this microscope is to be found under the number E 36—1905 in the Court Collection in the Museum.

In 1746 Adams produced his portable compound microscope, which was the first of its kind. Fig. 5 is a photograph of a later form of Adams' pocket compendius, which is scarce.

Another pocket microscope, known as Gould's "pocket microscope," was made

by Cary about 1828. A fine example of this beautiful little instrument was presented by Mr T. Court to the Museum and can be seen under the number E 15.

Just at this time Adams published his *Micrographia Illustrata or the Knowledge of the Microscope Explained*, a work which greatly stimulated microscopy and was the first work of its kind in this country.

We have now entered on a period fertile both in alterations of the microscope and in discoveries made by its agency. We have amongst the former the results of the labours of Adams, Martin, Baker and Dellebarre; and amongst the latter the investigations of Trembley, Ellis, Baker, Adams, Hill, Swammerdam, Lyonet, Needham and Withering, and to keep the pot boiling, so to speak, Dr Hill's treatise on *The Construction of Timber explained by the Microscope* published in 1770, which created a great sensation at that time.

During this period Adams invented a microtome for cutting thin sections of wood, which was improved by Cummings. With Cummings' microtome Quekett records that Custance cut unrivalled sections.

We must now rapidly survey three designs which bring to a close the non-achromatic period. The first of these is the "simple and compound microscope" by Martin, made about 1765. On it is engraved "B. Martin London invt. et fecit No.1,' which can be seen on the instrument lent to the Museum by Mr E. M. Nelson. It has no inventory number. It is of the universal type, i.e. can be used as a simple or compound instrument and is fitted with movement for aquatic objects and Lieberkühns for opaque objects. The last compound microscope with aquatic movement was made by Powell in 1840. The interesting feature in the design is the revival of the rackwork coarse adjustment. The teeth are cut on the inner side of a slotted bar, the opposite and plane side of the slot serving as a bearing surface for the block carrying the stage. A pinion attached to the block and engaging with the teeth is thus able to move the stage up or down. This block also slides upon a vertical rod of square section to which it can be clamped and which forms part of the fine adjustment. The rod is supported at its lower screwed end by threading a thumb screw and at its upper rounded end by engaging with a hole in a plate projecting sideways from the top of the limb. Thus when the thumb screw is turned in its collar, which is fixed to the pillar of the foot, the square rod being prevented from turning moves axially and carries with it the block bearing the stage. So the fine adjustment had to be unclamped before the coarse could be used, which appears to us a clumsy arrangement. The instrument also carries a superstage.

The second design is that exhibited in the "universal microscope" made by Bleuler about 1788 and forms an important link between Martin's and Jones' "most improved" which I shall describe shortly.

An example of Bleuler's instruments presented by Mr Court to the Museum bears the inventory mark E. 147—1911. This type of instrument was also made by Gregory and Wright as well as Shuttleworth, but the actual inventor is not known.

It has a folding tripod foot with a compass joint at the bottom of the limb, originally due to Martin. The limb is of △ section, at the back of which is placed

the rack, and the sliding block carrying the stage has a bearing in it for the pinion. Numbers upon the limb correspond to the objectives which are mounted in a disc. This disc form of objective changer is said to be due to Martin, and was an improvement upon the loose sliding changer. There is no fine adjustment, the rack being sufficient. The body attached to the end of a horizontal arm can be moved in arc or antero-posteriorly, and can be entirely removed so that with the disc only it can be used as a hand glass. The condenser and mirror are fitted to slide up and down the limb.

The microscope has now evolved into an optical bench with the stage condenser and mirror movable along it, while at one end the tube is fixed, and the other end bears a joint to enable the whole device to be inclined.

The third and last design was called Jones' "most improved" compound microscope made in 1798, and a model made by Dollond and lent by Mr Court is to be seen in the Museum. This type of microscope represents the culminating point reached by the dioptric instrument before the introduction of the achromatic objective and was in use several years after that took place. The model referred to also dates from 1798 and resembles in general design the Bleuler type.

The limb however which is □ in section is hinged at its upper third to the top of a vertical pillar rising from the folding tripod. The lug which forms part of this hinge necessitates an alteration in the design of the coarse adjustment of the stage, and it will be seen that the rack is placed on the front part of the limb. The mirror is mounted in the simple modern manner. Here again there is no fine adjustment, as in all probability the well cut rack is sufficient for this purpose, a lesson which should be taken to heart by some of the modern makers. Good construction frequently replaces unnecessary complication.

During this time and up to 1824, when the invention of the achromatic objective marked a new and more scientific era in microscopy, all improvements were mechanical and the workmanship was at its zenith. Chevalier lamented the fact that "England was more laborious than France and appeared to have the monopoly of the manufacture of the best instruments."

Just previous to the introduction of achromatism men were getting dissatisfied with the poor definition of the compound and the limited power of the simple instrument. So we find Amici, having failed in his attempts to produce achromatic lenses, turning his attention to Newton's suggestion of a catoptric instrument and producing in 1815 his famous reflecting microscope. A model of this, made about 1830 for Wollaston by Professor Amici, who was at that time Director of the Observatory at Florence, can be seen in the Museum, labelled E 6—1881. The reflector is ellipsoidal with an aperture of about 1″ and a focal length of 2″. Newton's suggestion had been acted upon, however, at a very much earlier date than this, i.e. in 1736 by Barker* and in 1738 by Smith who produced perhaps the most perfect of the catoptric forms, which was really a short focus Gregorian telescope. The difficulties of construction and illumination of the object in these inventions soon stifled their inception.

* *Phil. Trans.* vol. 39, pp. 259–261.

Brewster now points out that the only hope is to use the single lens of a material with much higher index of refraction and lower dispersion, he himself making lenses of ruby and garnet and finding that they were superior to the glass lenses. At the instance of Goring in 1824 Pritchard constructed diamond lenses. He also made subsequently lenses of ruby, garnet, and sapphire.

In 1812 and also in 1829 Wollaston produced his famous doublets. Indeed from 1812 to 1832 great attention was paid to the design of doublets and those who attacked the problem include such famous names as Brewster, Sir John Herschell, Coddington and Holland.

The time was now getting ripe for the advent of achromatism, for as early as 1762 Euler began to discuss the theory of achromatic microscopes. In 1747 he suggested achromatic object glasses for telescopes, and Chester More Hall in 1729 is said to have constructed at least two achromatic telescopes. In 1784 Aepinus made attempts to achromatise the microscope but failed.

We must however pass over all these early and historical attempts to improve the optical performance of the instrument, and bring ourselves to a halt at the year 1824, which marks the starting point of more exacting and more rigid mechanism in the construction of the microscope. In this year, on April 5th, Selligues on the Continent presented to the Académie des Sciences a microscope made by Chevalier with achromatic lenses, and it will be noted, if the illustration on p. 148 of Carpenter's *Microscope*, 1901, be consulted, that the whole construction is more robust though somewhat similar to the design of Jones' "most improved" instrument. Fresnel's report on the optical performance of this instrument was not altogether favourable.

In the same year, at the suggestion of Goring, Tully, without knowledge of what was being done on the Continent, made an achromatic objective. "Mr Lister," says Quekett, "who was engaged with Mr Tully in perfecting the achromatic object glass, finding that all the microscope stands hitherto made were not sufficiently steady for the use of high powers, directed his attention to the improvement of this part of the instrument." This was Joseph Jackson Lister—the father of Lord Lister of immortal fame—a man whose mind teemed with invention. He employed James Smith in 1826 to construct to his own working drawings the microscope, a model of which, lent by Mr Nelson, may be seen in the Museum—a truly historical instrument. The body tube is screwed to a solid limb and is moreover strutted to the base to secure the necessary rigidity. The optical tube slides within the body tube and is actuated by rack and pinion. There is no fine adjustment. The stage has rectangular movements actuated by milled heads, but the tripod foot remains unimproved. Here the early idea of movable body and fixed stage, contrary to that exhibited by instruments of the latter part of the 17th century and Selligues' design, is reintroduced to remain, as the best conceivable arrangement, up to the present time.

In 1841 Smith supplied the Microscopical Society with one of his newly constructed achromatic compound microscopes. The compound body of this instrument was mounted so as to slide in the groove of a strong bell-metal arm, the design of George Jackson—a plan adopted by Smith in all his large instruments.

This instrument is labelled E. 28—1891 in the Museum. This plan is well shown

in Smith and Beck's smaller achromatic, in which the groove in the limb is continuous with a similar groove in a smaller limb below the stage for the purpose of carrying the substage apparatus. Not only is this design excellent from the point of view of rigidity but also from the point of view of accurate and cheap production. It appears to have been forgotten, which is a pity. The fine adjustment is performed by turning a milled head which causes the nose-piece to slide in or out of the compound tube. This was a bad design which cannot give accurate axial movement to the objective, and the restraining screw being placed to one side of the axis of movement subjected the sliding piece to a couple which caused uneven wear. This form of fine adjustment is also seen in an earlier form of James Smith's microscopes which also embodied the Jackson limb. This instrument is of interest as it was one of three made by Smith which did not find favour in the market. The first model was sold to Richard Low Beck, Lister's partner in the wine trade.The second was sold to Alexander of Ipswich, and Lister bought the third model without objectives, etc.

Under the label 1919—469 in the Court Collection an example of the early design by Andrew Ross exhibiting the Jackson limb may be seen, and in many of these instruments the fine adjustment was of precisely the same design as in the two preceding instruments. It was just this deficiency of the fine adjustment for the Lister-Jackson type of limb, as it is now sometimes called, which caused Ross to try a host of various modifications. But from about 1841 he worked only at the long lever method as applied to the nose-piece through the "cross arm" and brought it to a high state of perfection.

The full possibilities of this method were pressed into service by Powell, and he made a microscope in 1841 with an extremely delicate fine adjustment applied to the stage, which was not an advance in the right direction. This instrument is in the Museum and was presented by Mr F. Hughes. The need that was felt at this time for rigidity is well exhibited in this model.

Great rigidity of the limb is also a marked feature of the modern Rosenhain Metallurgical Microscope. In this design the tube is fixed and the stage is movable by coarse as well as fine adjustments, conditions which are necessary for the special use for which the instrument is designed.

In the earlier models of Ross and Powell the pillar supporting the limb was single. The difficulty of making a sound knuckle joint and the want of rigidity of a single pillar caused Jackson in 1838 to introduce the use of two pillars between which the limb was swung on trunnions. This design is well shown in an early microscope made by Powell and Lealand shortly after P. H. Lealand, Powell's brother-in-law, became a partner with him, and this historical model is to be found in the Court Collection, labelled 1921—212. The manner of mounting the compound body was peculiar and was described by Powell in the *Transactions of the Society of Arts*, vol. 53, but as it never came into general use I shall not stop to describe it.

A historical detail of design however is exhibited in the mechanical stage of this model. In 1832 Hugh Powell was awarded a medal by the Society of Arts for his mechanical stage movements on the "Turrell system" which he first constructed

for Edmund Turrell. The "Turrell system" consists in placing the two milled heads of the rectangular movements side by side with a common axis. This method of mounting the two milled heads is extremely convenient in use especially when the microscopist has to carry out prolonged observations. A very beautiful application of the Turrell system is to be seen in Zeiss's detachable mechanical stage, illustrated on p. 41 of Spitta's *Microscopy*.

Jackson's method of support by means of two pillars could not be so rigid as a solid structure embodying both foot and support. This further advance in design was achieved by Ross and Powell in the form of their well-known characteristic tripods.

In Powell and Lealand's No. 1 stand the fine adjustment of Ross which originally was a lever of the second order is modified by Powell into one of the first order. This adjustment is conveniently housed in a hollow box into one end of which the body screws, the other end being fixed to a solid prism. The solid prism, to which a rack is fixed along its length, fits the lower hollow portion pivoted to the stand. This form of limb, characteristic of the Ross and Powell microscopes at this time, is known as the "bar limb"; this form and the Lister-Jackson limb are the two modern types in this country.

In the meantime rapid developments were being made on the Continent by such makers as Plossel and Schiek of Vienna, Pistor of Berlin, and Chevalier, Oberhauser and Nachet of Paris. The early microscopes of Schiek and Pistor more especially were somewhat similar to Ross's early model with the Jackson limb, and Chevalier's microscopes were peculiar for the horizontal mounting of the tube which carried a reflecting prism at the objective end, so that the optical axis of the objective should be vertical. The stage had coarse and fine adjustments. We have hardly time to notice these microscopes, but the forms of Oberhauser and Nachet appear to be the forerunners of the modern Continental limb or stand. Part of the limb is a hollow prism which slides over a solid one projecting vertically from the stage, and the fine adjustment is performed by turning a screw head which causes the limb to slide up and down the solid prism. This is an unsound design, for the whole weight of the compound body and limb, together with the coarse adjustment and all the twisting strains to which it may be subjected, are borne by the micrometric screw of the fine adjustment. Yet this design has persisted and is characteristic of the Continental microscopes.

In conclusion I would like it to be understood that some of the earliest dates which I have given are still matters of dispute, and that Prof. Govi's conclusions with regard to Galileo are not yet universally accepted. A faultless history, or indeed a correct partial history of the microscope, has yet to be written. In fact, the unbroken history of this remarkable instrument is not known. It has been quite impossible in the short time at my disposal to give any story of the intensely interesting details of the ordinary microscope, much less of the many varied and special forms which have evolved for all the world like living organisms that take shape and function in the crucible of environment. I have however tried to sketch a skeletal history of the mechanical construction of the microscope, and to indicate a few of the chief milestones on the road of invention.

My best thanks are due to the director of the Science Museum, Col. H. G. Lyons, F.R.S., for his kindness in lending me the historical instruments in the magnificent Court Collection as well as the other valuable models in the Museum which have been exhibited this evening. Also I cordially thank Mr T. Court, who with his unrivalled knowledge of these old instruments, has so kindly helped me to set them up correctly with histological preparations so that their optical as well as their mechanical performance may be observed and compared. In addition I am indebted to the kindness of Col. H. G. Lyons who has allowed me to use the blocks made from the five unique photographs which illustrate the paper.

REVIEWS

Durability of Optical Glasses. Report of an Investigation carried out by T. HAIGH, B.A., A.R.C.Sc.I., A.I.C. Pp. 51 + 10 plates. (London: British Scientific Instrument Research Association, 26, Russell Square, W.C. 1, 1921.) Price 7*s.* 6*d.*

Considerable attention has been paid during recent years to methods of testing the durability of glass. While to a large extent investigations on this subject have had for their object the testing of scientific and laboratory glassware, the problem is of considerable interest also to the optician to whom a knowledge of the weathering properties of the glasses he is using is almost as important as that of the optical properties.

The report gives the results of researches undertaken to determine whether simple tests could be devised by which the durability of different types of optical glass could be quickly ascertained without awaiting the results of experience by actual use over an extended period. Forty-seven glasses of Continental and British manufacture, comprising 35 different types whose optical constants are given, have been examined by various methods, including the iodeosin test of Mylius, the autoclave test, tests based on powdered glass, tests on immersed plates and the dimming test. These methods are described in detail and the results obtained by them for the various glasses are tabulated and analysed. The method adopted by at least one of our optical glass manufacturers of determining the amount of alkali dissolved by water at 80° C. from powdered glass does not seem to have been considered.

The behaviour of polished surfaces of the glasses under various methods of treatment is studied in detail and a series of 19 extraordinarily good photomicrographs of autoclaved glass surfaces are given which are extremely instructive and suggestive. From the examination of a series of simple glasses a few general deductions are drawn with regard to the influence of composition on the alkalinity of the surfaces.

The results of the investigation lead the author to the conclusion that it is not possible to suggest any simple tests which are thoroughly reliable and which will not lead to wrong conclusions. At the same time, however, he shows that it is possible to make a reasoned and fairly reliable estimate of the probable durability of a glass under the conditions for which it is likely to be used.

The report is worthy of careful study by those interested in the manufacture of glass and also by opticians who may require to use new or untried types.

F. F. S. B.

Process Work and The Printer. Edited by WILLIAM GAMBLE, F.R.P.S., F.O.S. (London: Percy Lund, Humphries and Co., Ltd.) Price 6*s.* per annum or 1*s.* 6*d.* per quarter.

This quarterly, which contains a number of interesting short articles and specimen plates, questions and answers on process and printing problems, reviews of books, and lists and abstracts of patent specifications, is to be recommended to all who are interested in the technique of process work and printing.

J. S. A.

ABSTRACTS OF PATENT SPECIFICATIONS

178175. Morehouse, W. H. VEHICLE LAMPS.

An anti-dazzle fitting consists of a rectangular glass plate formed with vertical corrugations upon its back face, and bevelled at the edges. It is carried by a sheet-metal mounting held in place across the front of the lamp by a ring and bezel.

178195. Rogers, F. H. ILLUMINATED EFFECTS.

An illuminated motion effect, simulating smoke, rain, or falling water, etc., is produced upon a stationary translucent picture by interposing moving and fixed screens, partly opaque and partly transparent, between the picture and a source of light. In the form described, which produces the effect of a cataract, the lamp is surrounded by a cylindrical screen having inclined transparent and opaque stripes, which is rotated by the hot air from the lamp acting upon a fan. Between this and the picture there is placed a second screen, which is translucent with transparent spots or bars.

178290. Taylor, W. REFRACTOMETERS.

In a refractometer of the Abbe type the observing telescope is fixed to the reading microscope, and the graduated arc, which is moved by a screw, is attached to the pivot of the prisms. Compound prisms for correcting and measuring dispersion are carried by a sleeve, which is rotatable in its housing. Special arrangements are made to secure complete circulation of the water within the jacket surrounding the object prisms.

178388. Ritchie, D. J. VEHICLE LAMPS.

The central part of the reflector is ellipsoidal, and merges into a conical outer zone. The front lens receives the light from the outer zones of the reflector mainly upon its central portion, and from the central zone chiefly upon its marginal portions. The projected beam has a region of lesser illumination outside the main concentrated beam.

178415*. Optique et Précision de Levallois. BINOCULARS.

The two tubes are hinged to the side edges of a rectangular plate, so that they can be adjusted for interocular distance, or folded closely together. Apertures in the plate render it deformable, so as to secure parallelism of the axes.

178430*. Ruyter, H. de. PROJECTING OR VIEWING APPARATUS.

An illuminating window and either a projecting or a viewing lens are arranged at opposite sides of a cylindrical body, upon which are also mounted two magazines for receiving transparencies. A rotary drum receives one picture at a time, and carries it into position in front of the window. The apparatus is reversible, for use in either direction.

178436*. Optique et Précision de Levallois. EYEPIECES.

Divergent eyepieces for use in telescopes or Galilean binoculars comprise a positive lens, and a negative lens of higher power but smaller dispersion, placed at a distance equal to its focal length from the positive lens. Single or cemented lenses may be used, the eyepiece being corrected for spherical and chromatic aberrations, and giving a wide field of view.

178522. Terry, A. E. VEHICLE LAMPS.

A number of short tubes, or corrugated strips, preferably of dull-blacked metal, are aggregated within a clamping band, so as to provide a number of substantially circular passages for the light, thus constituting an anti-dazzle device. The Provisional Specification states that the tubes may be of oval, square, or other section, and highly polished inside.

* Specification published before acceptance.

178635. Morris, F. H. VEHICLE LAMPS.

The source of light is placed at the focus of a parabolic reflector forming the upper part of the lamp, and directing the light downwardly. In the lower part of the lamp there are placed two reflectors, the forward one consisting of an inclined plane mirror which projects a beam forwards, while the other is parabolic in form, and confocal with the upper reflector. The light falling upon it is thus re-focussed upon the source.

178678. Hill, A. V., and others. POSITION FINDERS.

At two points of observation there are placed ruled surfaces, upon which the apparent positions of an object in space are noted, observing either directly through the surfaces, or by reflection therein, by means of aperture eyepieces. Formulae involving the constants of the system and the co-ordinates of the image upon the ruled surfaces enable the position of the object to be computed, and its course followed.

178724. Harwix, E. PRISMATIC BINOCULARS.

The body-work of each tube is formed in two transverse sections, each half having a cavity, in which one of the prisms is fitted. The two parts may be clamped directly together, or a diaphragm-plate may be interposed, into which the eyepiece tube and object-glass mount may be screwed.

178825. Pathé Cinéma &c. OPTICAL TOYS.

A box is divided by partitions into three compartments, of which the central one is fitted with a viewing lens at the front, and has an aperture filled with ground glass at the back. A flexible band of transparencies is passed across the sight aperture by means of a feed roller, and is caused to curl up in the end compartments, which thus serve as magazines.

178941. Tate, A. O. REDUCING GLARE; MATCHING COLOURS.

Rotary discs having slots or apertures cut therein in a special manner, so as to appear to transmit a uniform illumination, are employed to reduce the light or other rays reaching the eye from furnaces, crucibles, or glaring lights, and thus to facilitate their examination. Similar discs painted with stripes of colour are utilised for blending colours, to form a definite tint.

178978. Pollard, A. F. C. SPHEROMETERS.

The body to be tested, for example, a lens, is maintained in fixed relationship to a sphere of known radius, while a glass plate is caused to roll upon them both without slip. The relative lengths of the arcs of movement are ascertained by observing the shift of the Newton-ring systems which form around the points of contact, and the curvature of the lens at that point is thus determined. Two forms of apparatus are described. In the first form the glass plate, which may be partially counterbalanced, rests with one end upon the spheres, and is supported at the other end by a carrier-arm borne by a float upon mercury. The rolling of the plate is produced by varying the level of the mercury by means of a plunger. The second form of apparatus includes swivelling carriers for the lens and standard sphere, by which any part of the lens can be brought into contact with the plate, so that the curvature near any point can be ascertained.

179082. Beck, C. MICROSCOPE CONDENSERS.

A differentially-threaded mount serves to adjust the position of the lower member of the condenser, and at the same time opens a gap in the mount shell corresponding to the thickness of the slide for which the condenser is then properly adjusted for immersion working.

179139*. McNair, G. H. S., and others. PROJECTION APPARATUS.

It is proposed to use a divergent (bi-concave) flint lens placed between the transparency and the source, to project an image upon a translucent screen carried by a sliding base. Change of picture is effected by a carrier wheel, which is intermittently rotated to receive a slide from an upper magazine, and after displaying it, to transmit it to a lower magazine. The slides pass through the lower magazine, and are returned by a carrier-wheel at the back again into the upper magazine.

179232. Stafford, F. VEHICLE LAMPS.

A number of tubes forming an anti-glare device, and made of glass or celluloid, are secured to a ring, which is adjustably mounted upon the lamp body, so that the light may be directed at any desired inclination.

179281. Gibson, H. C. VEHICLE LAMPS.

A projecting mirror is placed behind the bulb, and a reflector above it, extending down the sides of the lamp if necessary, so as to throw the light down upon the road. Two cylindrical lenses, with their axes vertical, spread the light horizontally. The source of light is arranged to be movable vertically by means of a Bowden mechanism, to give a downcast beam when desired.

179491. Ritchie, D. J. LAMP LENSES.

One or both faces of the projecting lens are formed with vertical corrugations, to extend the beam laterally, without producing striations. To deflect downwardly the rising light the upper edge of the lens may be bevelled off, and its lower edge recurved, or horizontal prisms may be fitted to these parts. Either surface of the lens may be corrected aplanatically.

179629. Ensor, H. E. VEHICLE LAMPS.

A hinged section, normally forming part of the reflector, and lying below the bulb, can be tilted by suitable connections so as to throw the light upwardly. At the same time a hood is brought down to obscure the upper part of the face of the lamp.

179817. Barr and Stroud, Ltd., and another. MEASURING SMALL ANGLES.

The instrument is intended to measure the angular subtense of a distant object, such as a ship, so that when its actual length and the range are both known or assumed, a change of course can be ascertained. Two telescope systems, one above the other, are directed transversely to the line of sight, a single entrance prism being sufficient. The eyepiece is directed obliquely downwards towards the separating prism, which consists of two members in contact, one half of the common surface being silvered before the parts are cemented together. The divided field so produced is brought into the necessary coincidence by refracting prisms moving in the beam, their positions, as shown on suitable counting drums, giving the measure of the angle desired.

179849. Morsbach, K. CONDENSER LENSES.

Four components are employed, the two nearest the source being concave meniscus lenses, preferably of hard glass, placed close up so as to collect a wide cone of light. The other two components may be plano-convex.

179901. Sutcliffe, J. H. OPTICAL APPARATUS.

An optical arrangement for producing a double image, applicable to ophthalmometers, rangefinders, etc., includes a medial prismatic element and a marginal prismatic element, which are relatively rotatable. Two portions of a decentred lens, each working in conjunction with a prismatic element, may be employed.

* Specification published before acceptance.

179902*. Optische Ansalt, C. P. Goerz A. G. GALILEAN BINOCULARS.

The object glasses are mounted upon a carrier which is moved by a central operating-screw for the purpose of focussing, and the sight apertures are fitted with cover glasses. The eyepieces are mounted in separate casings, which are beaded on to the main body.

179946*. Meyère, A. ARC LAMPS.

The carbons are arranged at right angles to each other, and are fed by clamps moving along screwed shafts, which are rotated at different rates, so as to compensate for the unequal rates at which the carbons are consumed when direct current is employed. A worm and sector is provided to swing the arc as required, and a fore and aft adjustment is also provided.

179948*. Kodak, Ltd. PROJECTION APPARATUS.

In a vertical projection apparatus, adjustable for magnification, the position of the object as read off upon a scale gives the magnification, and a cam or ramp of suitable design adjusts the lens so as to maintain the image always in focus. An illuminating device with a reflector and diffusing screens may be fitted behind the object negative to give uniform illumination.

179972. Piper, F. G. VEHICLE LAMPS.

The bulb is adjustable in position axially within the reflector by mounting it in a tube, which can be drawn back by a Bowden wire mechanism, against the action of a spring.

180028. Barr and Stroud, Ltd., and another. RANGE FINDERS.

Forms of eyepiece prisms are described, suitable for use in coincidence rangefinders, and arranged to give laterally correct and erect images.

180291*. Corbetta, G. PROJECTION APPARATUS.

The beam is condensed by an ellipsoidal reflector, which may, however, be modified towards a spherical or parabolic form to secure uniform illumination. One of the arc carbons passes through the centre of the mirror, towards which the crater of the arc is presented. The necessary angular and focussing adjustments are provided.

180407. Court, J. R., and another. VEHICLE LAMPS.

An anti-glare front glass is plane upon one face, but the other is shaped to provide a series of horizontal prisms, which decrease in height, but increase in angle from the top downwards. Lateral spreading of the light is secured by making the prisms concave about a vertical axis. In the base of each prism there is formed a recess, triangular in both sections, the faces of which are ribbed to diffuse the light.

180618. Moon, S. H., and another. PROJECTION APPARATUS.

A film bearing transparencies for projection is caused to have a spherical or bi-curved form when in position in the lantern, either by permanent moulding of each object portion, or by placing in the gate a transparent shaping-piece, preferably not of glass, so that it shall not be affected by the heat.

180629*. Bertron, E. DAYLIGHT PROJECTION-SCREENS.

A chamber having a series of partitions with sight apertures formed therein is mounted behind the screen, which is viewed from the back. A similar chamber may be placed between the lantern and the screen.

* Specification published before acceptance.

180656*. Keller-Dorian, A. THREE-COLOUR APPARATUS.

The dimensions of the optical system employed for viewing or projecting colour films embossed with microscopical refractive elements are based upon the size of these elements upon the film. The optical system includes a projecting lens, and two lenses behind the film, which may act either as a condenser, or as a magnifying system for direct viewing.

180731. Schumann, R. GRINDING LENS EDGES.

The lens is held between rubber discs at the opposed ends of aligned shafts, and thereby rotated in contact with a grinding wheel. A series of templets corresponding to the lens shapes most commonly required co-operate with a guide roller, whereby the lens-carrier is moved relatively to the grinding-wheel, so as to acquire the shape of the templet selected. A cylindrical cam with an adjustable throw gives the shaft an axial oscillation, to prevent the formation of grooves in the edge of the grinding-wheel.

180805. Culver, E. VEHICLE LAMPS.

An annular toric lens surrounds the lamp bulb, and a conical reflector projects the emergent light forward through the lamp front, which may carry cylindrical or other lenses to give the distribution required, in addition to a central spherical or spherically corrected lens. A dished reflector is placed behind the bulb.

180949. Warmisham, A., and another. LENSES.

To increase the relative amount of light passing through from the edges of the field, lenses of the Petzval type are designed to have a back focal length not greater than ·35 of the equivalent focus.

181312*, 181314*, 181315*. Grossmann, H. MOULDING PLASTIC MATERIALS.

Two discs rotating edge to edge, and having cavities formed in the contacting surfaces, are employed in moulding plastic materials. The moulded body is expelled by a pin which is pressed outwardly by a fixed cam through an aperture in the base of the cavity at the appropriate moment. To ensure accurate registration, at least one of the discs is adjustable angularly upon its shaft. The pressure is regulated by a link coupling the shafts of the two discs, and provided with an eccentric bearing-bush at one end.

181347. Cushman, A. S. COLOUR EFFECTS.

In front of a projector there is placed a disc having opaque and coloured areas. The beam, thus rendered intermittent and of varying colour, impinges upon a moving object, such as a series of streamers in the draught from a fan, producing the effect of a large number of moving coloured sections. A vessel of water, which is shaken to produce moving drops, may be employed to constitute the illuminated body.

181409. Sapp, T. SIGNS.

The necessary lettering is formed upon a ribbed metallic or mirror-glass surface, which is then faced by a glass plate, ribbed upon one or both surfaces, in a direction transverse to the ribs on the back plate.

181434. Hailwood, E. A. MOULDING GLASS.

The necessary amount of glass is measured or gauged in a cup, which is then passed to the mould, where it is shaped by the pressure of a plunger. Finally, it is given an additional squeeze by a tube surrounding the plunger, so as to free the article from bubbles, and properly size it.

* Specification published before acceptance.

181467. Norton, D. E. BINOCULARS.

Field glasses are provided with a fitting which enables them to be secured to a walking-stick or umbrella, so as to hold them conveniently for use.

181579. Boulitte, G. C. SECURING FILAMENTS.

For the purpose of securing filaments in oscillographs and other apparatus, the holder or support terminates in a bead of soft metal, in the surface of which a groove is formed. The filament is laid in this groove, and secured by compressing the bead upon it.

181661. Seyty, A. SIGNS.

An electric lamp is placed centrally within a parabolic or other reflector, and bears suitable lettering in reversed characters, which are thus correctly seen after reflection.

181675. Naamlooze Vennootschap Philips' Gloeilampenfabrieken. ARC LAMPS.

Refractory electrodes are enclosed in a bulb containing an inert gas at such a pressure that connection to the source of supply produces a glow discharge, which then passes over into an arc discharge. The electrodes may be surrounded by a charged cage, so as to prevent deposition upon the walls of the lamp.

181708*. Geuns, J. E. C. van. VEHICLE LAMPS.

A series of thin parallel plates is so arranged as to intercept the rays of light passing in a direction liable to cause dazzle. The lower surface of the plates may be made reflecting, and the upper surface is darkened or tinted.

181782. Moon, S. H., and another. PROJECTION SCREENS.

Curved screens are built up in sections, which may be formed from wood or metal faced with plaster.

181804. Bendmann, H. ADVERTISING.

An optical projection apparatus is fitted with two obscuring-members, which act in conjunction so as to display the working upon a slide letter by letter and line by line. A number of slides are connected in a chain, the arrangements being such that, at the completion of the movements of the obscuring-members, a new slide is brought into position, and the parts are restored for a new operation.

R. J. T.

* Specification published before acceptance.

PROCEEDINGS

AT THE

MEETINGS OF THE OPTICAL SOCIETY

1921–22

13th October, 1921

Extraordinary Meeting held at the Imperial College of Science.

Mr R. S. WHIPPLE, M.I.E.E., President, in the Chair.

The following alteration of Rules proposed by the Council, of which the necessary notice had been given, was moved from the Chair and carried unanimously.

(1) A Member may at any time become a Life Member by payment of a composition of Twenty Guineas, provided that his subscription for the previous year has been paid.

(2) A Fellow may at any time become a Life Fellow by payment of a composition of Thirty Guineas, provided that his subscription for the previous year has been paid.

(3) An applicant for membership may, upon election, become a Life Member by the payment of a composition of Twenty Guineas in addition to the entrance fee.

(4) A Life Member, upon election as a Fellow, shall pay annually the difference between a Member's and Fellow's subscription or may compound for this additional subscription by a single payment of Ten Guineas.

The following resolution was moved from the Chair:

That the Resolution passed by the Council on the 11th March, 1915, suspending certain members, subjects of countries then at war with Great Britain, be now revoked.

13th October, 1921

Meeting held at the Imperial College of Science.

Mr R. S. WHIPPLE, M.I.E.E., President, in the Chair.

The death of Mr William Shackleton, a Vice-President, was announced.

The Thomas Young Oration was delivered by Dr Charles Sheard, Editor of the *American Journal of Physiological Optics*, on the subject "Some Important Physical and Physiological Relationships between Radiant Energy and the Visual Apparatus and Processes."

10th November, 1921

Meeting held at the Imperial College of Science.

Mr R. S. WHIPPLE, M.I.E.E., President, in the Chair.

The following papers were read:

"The Interocular Distance." By JAMES WEIR FRENCH, D.Sc.

"Note on the Thin Astigmatic Lens." By THEODORE CHAUNDY, M.A.

"The Path of Rays in Periscopes having an Inverting System comprising Two Separated Lenses." By Dr ALEXANDER GLEICHEN. (Translated and read by Dr French.)

24th November, 1921

Special Meeting held at the Imperial College of Science.

Mr R. S. WHIPPLE, M.I.E.E., President, in the Chair.

Prof. F. J. CHESHIRE, C.B.E., gave a demonstration of Polarisation Apparatus, illustrated by examples from the Science Museum collection, specially loaned for the occasion through the courtesy of the Director, Col. Lyons, F.R.S.

8th December, 1921.

Extraordinary Meeting held at the Imperial College of Science.

Mr R. S. WHIPPLE, M.I.E.E., President, in the Chair.

The following proposed form of Rule VII to replace the previous rule was moved from the Chair and carried unanimously, due notice having been given to all members:

Any Society, Partnership firm, Company, Government Department, or other corporate body may be admitted as a member under the conditions specified in Rule III. The entrance fee shall be £1. 1*s*. and the annual subscription £3. 3*s*. Each such body may nominate one delegate to exercise its rights as a member of the Society. Corporate membership shall entitle such body to one copy of the Society's Transactions.

8th December, 1921.

Meeting held at the Imperial College of Science.

Mr R. S. WHIPPLE, M.I.E.E., President, in the Chair.

The following papers were read:

"An Auto Stroboscope and an Incandescent Colour Top." By F. L. HOPWOOD, D.Sc., A.R.C.Sc., F.Inst.P.

"Achromatic One-Radius Doublet Eyepieces." By Lt.-Col. J. W. GIFFORD, F.R.A.S.

"A Physical Study of Spherical Aberration." By L. C. MARTIN, D.I.C., A.R.C.Sc., B.Sc.

4th and 5th January, 1922.

Annual Exhibition of Electrical, Optical, and other Physical Apparatus held at the Imperial College of Science, jointly with the Physical Society of London.

The following discourses were given:

On 4th and 5th January: "The Johnsen-Rahbek Electrostatic Telephone and its Predecessors." By ALAN A. CAMPBELL SWINTON, F.R.S.

On 4th January: "Radium: its Application in Peace and War." By F. HARRISON GLEW.

On 5th January: "The Employment of Coarse Wire Gratings in Astronomy." By Sir FRANK DYSON, F.R.S.

12th January, 1922

Meeting held at the Imperial College of Science.

Mr R. S. WHIPPLE, M.I.E.E., President, in the Chair.

The following papers were read:

"The Barr and Stroud 100 ft. Self-contained Base Rangefinder." By JAMES WEIR FRENCH, D.Sc.

"The Manufacture of Optical Glass." By C. J. PEDDLE, M.B.E., D.Sc., F.I.C., F.Inst.P.

"The Optical Three Apertures Problem." By T. SMITH, M.A., F.Inst.P.

9th February, 1922

Annual General Meeting held at the Imperial College of Science.

Mr R. S. WHIPPLE, M.I.E.E., President, in the Chair.

The Treasurer's Report and Balance Sheet were presented by the Chairman in the absence of the Treasurer. It was suggested that in future years the stock held by the Society should be entered at its current value. The Report and Balance Sheet were then approved.

The Secretary presented the Annual Report of the Council, which was approved.

The Chairman read the list of persons nominated by the Council for office in the year 1922–23 and declared all these duly elected as no other nominations had been received.

President: Sir FRANK DYSON.
Vice-Presidents: Prof. F. J. CHESHIRE, Mr T. SMITH, Mr R. S. WHIPPLE.
Business Secretary: Prof. A. POLLARD.
Papers Secretary: Mr F. F. S. BRYSON.
Treasurer: Major E. O. HENRICI.
Librarian: Mr J. H. SUTCLIFFE.
Editor: Dr JOHN S. ANDERSON.

Council:

Already serving:	Elected, February, 1922:
Mr L. BOOTH.	Dr JOHN S. ANDERSON.
Mr W. GAMBLE.	Instructor-Commander T. Y. BAKER.
Mrs C. H. GRIFFITHS.	Mr R. W. CHESHIRE.
Mr J. GUILD.	Dr R. S. CLAY.
Prof. A. W. PORTER.	Dr J. W. FRENCH.
Dr MULLINEUX WALMSLEY.	Mr L. C. MARTIN.
	Mr F. TWYMAN.
	Mr A. W. WHITWELL.

The new President, Sir Frank Dyson, was then introduced by Mr R. S. Whipple and took the Chair.

Votes of thanks to the retiring Officers and Members of Council, to the Hon. Auditors (Mr H. A. Hughes and Mr A. B. Clark), and to the Governors of the Imperial College, with Prof. Callendar, for the use of the College premises for the meetings of the Society were unanimously passed.

Prof. A. A. Michelson and Dr Moritz von Rohr were elected Hon. Fellows of the Society.

9th February, 1922

Meeting held at the Imperial College of Science.

Sir FRANK DYSON, F.R.S., President, in the Chair.

The Chairman announced the death of Sir William Christie, an Hon. Fellow of the Society.

The following papers were read:

"The Structure of Abraded Glass Surfaces." By F. W. PRESTON, B.Sc., A.M.I.C.E.

"Variation in Refractive Index near the Surfaces of Glass Melts." By F. TWYMAN, F.Inst.P. and A. J. DALLADAY, A.Inst.P.

"The Stress Conditions surrounding a Diamond Cut in Glass." By A. J. DALLADAY, A.Inst.P., and F. TWYMAN, F.Inst.P.

"Achromatic One-Radius Doublet Eyepieces" (Supplementary Note). By Lt.-Col. J. W. GIFFORD, F.R.A.S.

9th March, 1922

Meeting held at the Imperial College of Science.

Sir FRANK DYSON, F.R.S., President, in the Chair.

The following papers were read:

"A Criticism of the Nodal Slide as an Aid in Testing Photographic Lenses." By T. SMITH, M.A., F.Inst.P. and JOHN S. ANDERSON, D.Sc., F.Inst.P.

"A Non-Polarising Spectrophotometer." By A. J. BULL, M.Sc., F.Inst.P.

"The Photometry of Optical Instruments." By J. GUILD, A.R.C.Sc., F.R.A.S., F.Inst.P.

"A Projective Treatment of the Submarine Periscope." By T. SMITH, M.A., F.Inst.P.

"Some Measurements of the Stresses produced at the Surfaces of Glass by Grinding with Loose Abrasives." By A. J. DALLADAY, A.Inst.P. (Read by Mr Twyman.)

6th April, 1922

Meeting held at the Imperial College of Science.

Sir FRANK DYSON, F.R.S., President, in the Chair.

The following papers were read:

"Diffraction Halos in Normal and Glaucomatous Eyes." By H. H. EMSLEY, B.Sc. and E. F. FINCHAM.

"The Effect of Changes of Surface Curvature at the Focus of an Astronomical Objective." By E. WILFRED TAYLOR. (Read by Dr Anderson.)

27th April, 1922.

Special Meeting held at the Imperial College of Science.

Dr R. S. CLAY in the Chair.

Prof. A. POLLARD delivered a lecture on "The Mechanical Construction of the Microscope from a Historical Standpoint." The lecture was illustrated by lantern slides and instruments loaned from the collection in the Science Museum by kind permission of the Director, Col. Lyons, F.R.S.

11th May, 1922

Meeting held at the Imperial College of Science.

Mr R. S. WHIPPLE, M.I.E.E., Vice-President, in the Chair.

Papers on "Motor Headlights," dealing more particularly with the optical problems involved, were read and demonstrations of headlights and anti-dazzle devices were given.

It was announced that the Discussion on "Spectacles and Spectacle Construction" had been postponed till next Session.

8th June, 1922

Meeting held at the Imperial College of Science.

Sir FRANK DYSON, F.R.S., President, in the Chair.

The following papers were read:

"Angle Comparators of High Precision for the Goniometry of Prisms." By J. GUILD, A.R.C.Sc., F.Inst.P., F.R.A.S.

"The Changes in Aberrations when the Object and Stop are Moved." By T. SMITH, M.A., F.Inst.P.

"The Classification of Optical Instruments." By T. SMITH, M.A., F.Inst.P.

"Note on Achromatism with One Glass." By T. SMITH, M.A., F.Inst.P., and L. M. GILLMAN, B.Sc.

"An Improved Subjective Test for Astigmatism." By HERBERT S. RYLAND.